

Dynamic Modeling and Analysis for Electricity-Gas Systems With Electric-Driven Compressors

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Abstract—The dynamic gas flow model with static electric-driven compressor (EDC) model has been widely studied in coordinated analysis of integrated electricity-gas system (IEGS). However, as a crucial coupling unit, the boost characteristics of the EDC change dynamically with gas state, which may lead to unstable operation mode and then affect safe operating of IEGS. Meanwhile, its nonlinear model greatly increases the difficulty of analysis. This paper proposes a reduced-order transfer function model that considers EDCs for an accurate and efficient dynamic analysis of the IEGS. Firstly, a general two-port model in the Laplace-domain is derived to jointly analyze the pipeline and EDCs dynamic characteristics. The model based on transfer function, explicitly gives the relation between the pressures and flows state at ports. On this basis, to concisely analyze state variations of crucial nodes, the multiple cascaded pipelines and compressors are aggregated and to form an equivalent model. Meanwhile, to reduce the computational burden of the high-order Laplacian “s” in the aggregated process, a reduced-order method is developed based on Jordan continued-fraction expansion by preserving the main gas dynamic feature. Then the analytical expression of state fluctuation can be conveniently derived to analyze the underlying impact and interaction between power system and gas system. Finally, case studies are conducted to prove the effectiveness of proposed model and reduction method.

Index Terms—Integrated electricity-gas system, electric compressor, dynamic analysis, transfer function, reduced order.

I. INTRODUCTION

THE INTEGRATED electricity and natural gas system (IEGS) has been widely developed to promote a

more efficient energy utilization and renewable accommodation [1], [2], [3]. The increasing number of coupling equipment such as gas-fired power generators (GPG) and electric driven compressors (EDCs) have enhanced the interaction and interdependent between the electricity power system (EPS) and natural gas system (NGS) [4]. For instance, the EDCs could significantly reduce emissions and operating costs, which brings additional opportunities for the integration of renewables in EPS [5], [6]. However, the intensive coupling and complex dynamic models challenge their combined analysis [7]. Firstly, the compressors used to compensate for pressure loss are essential equipment for the IEGS to maintain the safety of IEGS, but their dynamic characteristic is neglected in existing models. The characteristic not only directly changes the boost pressure ratio and gas state, but also leads to ill-situations such as surge and choke, significantly affecting the safe and reliable operation of IEGS [8]. Secondly, since the flow within the gas pipeline is described by partial differential equations (PDEs), the combined modeling of pipeline dynamics and compressor dynamics present challenges for simulation due to high complexity and non-analytical models [9]. Therefore, it is necessary to develop a new IEGS model to comprehensively describe system dynamics and accurately analyse the underlying impact.

To guide the combined analysis of IEGS, previous work has explored numerous transformation models of original nonlinear PDEs, which can be classified into three main categories: i) static models; ii) finite difference method based (FDM-based) models; and iii) (complex) frequency domain transformation based (FDT-based) models. For the static model, the Weymouth equation was introduced in [10] to describe the static gas flow. Then, the variable gas density and calorific value caused by multi-type gas supply sources were modeled into the Weymouth equation in [11] to improve the simulation accuracy. In [12], a new static equivalent model of NGS was proposed to formulate the gas consumption of compressors. It demonstrates that the compressors consume a relatively large proportion of the transmitted energy. Since the above models express the gas flow and pressure in a nonlinear and implicit form, which adds significant computation burden for the IEGS analysis, an analytical model was proposed in [13] by converting the Weymouth equation into the form of holomorphic function. The above studies describe the physical characteristics of NGS by using a set of algebraic equations.

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However, they overlook the dynamic variation of the gas state over time, leading to a mere snapshot of system operations and inevitable errors.

To capture the dynamic characteristics of NGS, different PDEs are modeled and numerically approximated by finite difference methods (FDM). In [14], a set of detailed gas transient models were discretized with implicit Euler difference method for coordinated scheduling of IEGS. Meanwhile, a linear static centrifugal compressor model was used to avoid the non-convex problem. In [15] and [16], the isothermal and non-isothermal gas flow models were developed for dynamic analysis of IEGS respectively. However, they ignore the role of the compressor. Literature [17] formulated a novel variable-coefficient gas flow model for IES analysis. The model not only poses the accuracy of the nonlinear model, but also significantly improves the convergence and stability. Then, a new cone-specific spatial branching strategy is developed to handle the non-convex nature of the integrated electricity-gas flow considering renewable energy sources [18]. To further efficiently analyze dynamic gas flow, a series of reduced network flow models were built in [19] and [20]. On this basis, dynamic compression protocols and time-dependent compressor ratios were proposed in [21] and [22] to construct a more comprehensive models. Since the accuracy and computation burden of the FDM methods depend on the difference step size, the temporal and spatial discretization criteria were proposed for NGS in [23] and [24] to select a reasonable step in analysis of IEGS.

The above studies successfully modeled the gas flow dynamic by using algebraic difference equations, which has a high robustness and ease of implementation. However, the embedded implications of system performance can hardly be obtained without closed-form expressions [25]. In addition, discretization in both time and space results in a substantial increase in grid points [26]. The variables at each point are dependent on the values at the preceding point, leading to a marked escalation in the dimensionality of equations. This, in turn, imposes a considerable computational burden.

To address those issues, several FDT-based models are proposed to analytically solve the PDEs. In [26], the energy circuit modeling based on Fourier transform was proposed to describe the gas flow dynamic. The relationship between pressure and mass flow were analytically modeled by equivalent lump branch parameters, which efficiently algebraized the original PDE models. However, this approach does not account for compressors. In [27], a generalized phasor method by using Fourier transform was proposed to model gas flow dynamics for IEGS dispatch problem, in which the compressor was treated as a short pipeline. However, the accuracy and efficiency based on Fourier transform depend on the number of superimposed components of different frequencies, and its essence is still a numerical solution [28]. Literatures [29], [30], [31] proposed the transfer function models to describe the dynamic gas flow based on Laplace transform for addressing the time-consuming simulation and complex implementation problems. These models are helpful for on-line applications of complex gas system. However, they simplified the equations under the assumption that the

gas flow speed was far smaller than the speed of sound, or maintained at a constant small value, which are not consistent with reality. In [25], an equivalent two-port pipeline model based on the first-order transfer function was proposed to reflect the gas flow dynamic by using Laplace transform. The model can directly express pressure variations with analytical equations and built the friendly interface to electric power, thus significantly improving the dynamic analysis efficiency of IEGS. Then, the model was developed in [32] to analyze the interaction between EPS and NGS in different time-scales. Furthermore, considering the privacy information protection between the independent entities of EPS and NGS, a multi-port model of NGS was proposed in [33]. However, the gas inertia term was ignored in above first-order transfer function for simplify analysis, which affected the accurate response with change in demand or supply. Moreover, they only studies the dynamic characteristics of pipelines, ignoring the gas compressors. Literature [34] proposed an electrical analogy model to describe dynamic natural gas flow, in which the static compressor model was analogous to a transformer in EPS and was also presented as a two-port model. Those efforts are capable of avoiding complex and time-consuming calculations.

Although extensive studies have been conducted on the model of NGS for combined analysis of IEGS, the following deficiencies still exist:

(1) The EDC model is often ignored or simplified into a static model with a constant boost ratio due to the complex nonlinearity. However, the boost ratio varies with the dynamic state changes in practice, which may cause the EDC to enter an ill operation mode. The simplified model ignores dynamic characteristics, which cannot express the actual operating boundary and lead to distinct errors that affect the accurate evaluation for the safe operation limits.

(2) In addition, the compressor couples the gas pipeline and electric network, but the effect of the compressor dynamic characteristics on the state of the two networks is not clear. There is a lack of the interface for joint analysis between the gas network dynamics and electric network key variables.

(3) The combined dynamic modeling of pipeline and compressor with strong nonlinearity will further increase analysis complexity. The numerical method without closed-form expressions require extensive repetitive computation, thereby introducing a huge computational burden. The analytical methods give the closed-form solutions, but will introduce high-order Laplacian “s”, which probably cause the model too complex to derive an analytical formula in the time domain.

To bridge the above mentioned gaps, this paper proposes a comprehensive framework for accurate and efficient dynamic modeling of IEGS. The main contributions of this paper are summarized as follows:

- 1) The dynamic characteristic of an electric-driven compressor is modeled as a two-port model for the first time, facilitating joint analysis of the relationship between gas flow dynamics and electric network key variables. Then, a general two-port model is proposed based on a transfer function. The model accurately describes the system's dynamics and provides an explicit expression.

- 2) The multiple cascaded pipelines and compressors are aggregated into an equivalent model based on identical physical variable of port model. Then, a reduced NGS topology with aggregated transfer matrices is presented. In this way, the variable relationships among crucial nodes could be obtained for a concise analysis.
- 3) A reduced-order method is developed to address the high-order Laplacian “s” encountered during the aggregating process by using Jordan continued-fraction expansion. The method is capable of preserving the main frequency dynamics, significantly reducing the model complexity while ensuring the accuracy. Then, an analytical model in the time domain encompassing the gas dynamics can be easily derived to describe the state fluctuations in IEGS.

The remainder of this paper is organized as follows: Section II introduces a transfer function model for NGS with electric-driven compressors to describe the dynamic characteristics. Section III offers the topology simplification strategy and reduced-order method to decrease the analysis complexity. Section IV describes the interaction between NGS and EPS based on proposed model. Sections V and VI give the case studies and conclusion, respectively.

II. TRANSFER FUNCTION MODEL FOR NGS WITH ELECTRIC-DRIVEN COMPRESSOR

A. Original Dynamic Gas Flow Model

We consider one-dimensional transient gas flow through horizontal pipelines under isothermal conditions [19], [23]. The dynamic behaviors of gas flow in pipelines are described by a set of governing PDE equations, shown as follows

$$\frac{\partial p}{\partial t} + \frac{4c^2}{\pi D^2} \frac{\partial m}{\partial x} = 0, \quad (1)$$

$$\frac{\partial p}{\partial x} + \frac{4}{\pi D^2} \frac{\partial m}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{8fc^2}{\pi^2 D^5} \frac{m|m|}{p} = 0, \quad (2)$$

where t and x are the time and space variables. p , m , ρ and u are the gas pressure, mass gas flow, gas density and gas velocity, respectively. c and f represent sound speed and friction factor, respectively. D is the pipeline diameter.

The third term $\partial(\rho u^2)/\partial x$ in Eq. (2) defines the dynamic pressure of the flowing gas [14], whose influence on the solutions of (2) is less than 1% under normal operating conditions. Thus, the term is usually omitted to simplify the description. Moreover, it is assumed that the gas flow directions do not change for pipelines with slow dynamic, and thus $m|m| = m^2$. Then, Eq. (2) is rewritten as

$$\frac{\partial p}{\partial x} + \frac{4}{\pi D^2} \frac{\partial m}{\partial t} + \frac{8fc^2}{\pi^2 D^5} \frac{m^2}{p} = 0. \quad (3)$$

B. Second-Order Transfer Function Model for Gas Flow

By approximating the non-linear term $\frac{m^2}{p}$ in (3) based on a reference point (m_0, p_0) [23], and then substituting deviation variables $\Delta m = m - m_0$ and $\Delta p = p - p_0$ from reference point into the variables m and p in Eq. (1) and (3), we can obtain

a set of linear partial differential equations. Then, taking the *Laplace Transform*, the Eqs. (1) and (3) are finally derived as

$$s\Delta p(s, x) + \frac{4c^2}{\pi D^2} \frac{\partial \Delta m(s, x)}{\partial x} = 0, \quad (4)$$

$$\frac{\partial \Delta p(s, x)}{\partial x} + \frac{4}{\pi D^2} s\Delta m(s, x) + \frac{8fc^2}{\pi^2 D^5} \times \left(\frac{2m_0}{p_0} \Delta m(s, x) - \frac{m_0^2}{p_0^2} \Delta p(s, x) \right) = 0, \quad (5)$$

where $\Delta p(s, x)$ and $\Delta m(s, x)$ are the pressure and mass flow in Laplace domain. By using 2nd accurate difference form $\frac{\partial X}{\partial x} \approx \frac{X_{i+1} - X_i}{\Delta x}$ and $X \approx \frac{X_{i+1} + X_i}{2}$ to discretize the spatial derivatives and term Δp , Δm , Eqs. (4)-(5) can be transformed to

$$s \frac{\Delta p(s, x_{i+1}) + \Delta p(s, x_i)}{2} + \frac{4c^2}{\pi D^2} \frac{\Delta m(s, x_{i+1}) - \Delta m(s, x_i)}{\Delta x} = 0, \quad (6)$$

$$\frac{\Delta p(s, x_{i+1}) - \Delta p(s, x_i)}{\Delta x} + \left(\frac{4}{\pi D^2} s + \frac{16fc^2 m_0}{\pi^2 D^5 p_0} \right) \times \frac{\Delta m(s, x_{i+1}) + \Delta m(s, x_i)}{2} - \frac{8fc^2 m_0^2}{\pi^2 D^5 p_0^2} \frac{\Delta p(s, x_{i+1}) + \Delta p(s, x_i)}{2} = 0. \quad (7)$$

For a two-port pipeline with length L_{ij} and the observation points set at ends i and j , the items $i + 1$ and Δx in (6)-(7) can be replaced by j and L_{ij} , respectively. Then, the matrix form expression of (6)-(7) can be given by

$$\begin{bmatrix} \Delta \Pi_j \\ \Delta M_j \end{bmatrix} = \begin{bmatrix} G_1^{i \rightarrow j} & G_2^{i \rightarrow j} \\ G_3^{i \rightarrow j} & G_4^{i \rightarrow j} \end{bmatrix} \begin{bmatrix} \Delta \Pi_i \\ \Delta M_i \end{bmatrix}, \quad (8)$$

where $\Delta \Pi_j$ and $\Delta \Pi_i$ are the expressions of $\Delta p(s, x_j)$ and $\Delta p(s, x_i)$ on L-domain, similarly for ΔM_j and ΔM_i . $G_{1,2,3,4}^{i \rightarrow j}$ are the transfer functions of pipeline $i \rightarrow j$, derived from (6)-(7) and expressed as

$$G_1^{i \rightarrow j} = -1 + \frac{k_1 + \tau_0}{\tau_2 s^2 + \tau_1 s + \tau_0}, \quad G_2^{i \rightarrow j} = \frac{k_2 s + k_3}{\tau_2 s^2 + \tau_1 s + \tau_0}, \\ G_3^{i \rightarrow j} = \frac{k_4 s}{\tau_2 s^2 + \tau_1 s + \tau_0}, \quad G_4^{i \rightarrow j} = -1 + \frac{2\tau_0}{\tau_2 s^2 + \tau_1 s + \tau_0}, \quad (9)$$

where τ_i and k_i are the coefficients, calculated in terms of pipeline parameters and operational conditions, shown as follows:

$$\tau_0 = -4DA^2 p_0^2 + fc^2 L_{ij} m_0^2, \\ \tau_1 = fAL_{ij}^2 m_0 p_0, \quad \tau_2 = \frac{DA^2 L_{ij}^2 p_0^2}{c^2}, \\ k_1 = -4DA^2 p_0^2 - fc^2 L_{ij} m_0^2, \\ k_2 = 4DAL_{ij} p_0^2, \quad k_3 = 4fc^2 L_{ij} m_0 p_0, \\ k_4 = \frac{4DA^3 L_{ij} p_0^2}{c^2}. \quad (10)$$

By using the presented second-order transfer functions model, the relationships in Laplace domain between pressures and flows at the ends of a pipeline can be explicitly expressed.

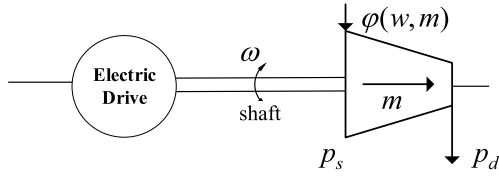


Fig. 1. The schematic of electric-driven compressor.

Remark 1: Compared with the traditional transfer function model of gas system, the model in this paper does not require the assumption of constant gas flow speed, and extra consideration is given to the gas inertia term in (5), which is helpful to capture slow dynamic information to obtain more intact results.

It is worth noting that the selection of reference point directly affects the approximation accuracy of Taylor expansion in (3). When there are significant changes in the system, a new operating point close to the current condition is typically chosen, and the Taylor series is re-expanded to reduce error.

C. Dynamic Model for Electric-Driven Compressor

In realistic gas networks, the compressors are necessary equipments to maintain the level of pressure and satisfy the demand of consumers. Moreover, for electric networks, the electrification of natural gas pipeline compressors can offer increased opportunities for integrating renewable energy sources [5]. This paper introduces the dynamic characteristic of the electric-driven centrifugal compressor. The schematic is shown in Fig. 1. The boost characteristic of the compressor is determined by the rotational velocity of compressor shaft w and the mass flow m through the compressor [35], which is expressed as

$$\varphi(w, m) = \left(1 + \frac{c_1 w^2 - c_2 m w - c_3 (w - c_4 m)^2 - \mu_G m^2}{c_p T_{GC}} \right)^{\frac{\kappa}{\kappa-1}}, \quad (11)$$

where μ_G is the friction constant of natural gas. T_{GC} is the gas temperature at the compressor suction side. c_p is the specific heat at constant pressure. κ is the specific heat ratio of natural gas. c_1 , c_2 , c_3 and c_4 are the operational coefficients, related to the number of blades, impeller radius, rotor blade angle, mean inducer radius, and blade outlet angle of compressor, respectively, which changes with different compressor in practice.

Based on the boost characteristic of compressor, we have the compressor's discharge pressure at time t

$$p_{d,t} = \varphi(w, m) \times p_{s,t}, \quad (12)$$

where $p_{s,t}$ is the suction pressure of compressor at time t . It is assumed that the angle velocity of compressor shaft keeps at a desired speed. Then, the discharge pressure is a function related to mass flow and suction pressure, which is rewritten as

$$p_{d,t} = \lambda_0 \left(\lambda_1 m_{s,t}^2 + \lambda_2 m_{s,t} + \lambda_3 \right)^{\frac{\kappa}{\kappa-1}} \times p_{s,t}, \quad (13)$$

where $m_{s,t}$ is the suction mass flow of the compressor at time t . λ_0 , λ_1 , λ_2 , and λ_3 are the parameters derived from (11), shown as

$$\lambda_0 = \frac{1}{(c_p T_{GC})^{\frac{\kappa}{\kappa-1}}}, \quad \lambda_1 = -c_3 c_4^2 - \mu_G, \quad (14)$$

$$\lambda_2 = (2c_3 c_4 - c_2)w, \quad \lambda_3 = (c_1 - c_3)w^2 + c_p T_{GC}. \quad (15)$$

It can be seen that the boost pressure characteristic is highly nonlinear with regard to the variables $p_{s,t}$ and $m_{s,t}$. To unify the analysis with the pipeline by using transfer function, we linearize the boost model (13) around initial points $m_{s,t-1}$, $p_{s,t-1}$. Then, the first-order Taylor series expansion of (13) is shown as:

$$\begin{aligned} p_{d,t} &\approx \lambda_0 \left(\lambda_1 m_{s,t-1}^2 + \lambda_2 m_{s,t-1} + \lambda_3 \right)^{\frac{\kappa}{\kappa-1}} p_{s,t} \\ &\quad + \frac{\kappa}{\kappa-1} \lambda_0 (2\lambda_1 m_{s,t-1} + \lambda_2) \\ &\quad \times \left(\lambda_1 m_{s,t-1}^2 + \lambda_2 m_{s,t-1} + \lambda_3 \right)^{\frac{1}{\kappa-1}} p_{s,t-1} m_{s,t} \\ &\quad - \frac{\kappa}{\kappa-1} \lambda_0 (2\lambda_1 m_{s,t-1} + \lambda_2) \\ &\quad \times \left(\lambda_1 m_{s,t-1}^2 + \lambda_2 m_{s,t-1} + \lambda_3 \right)^{\frac{1}{\kappa-1}} p_{s,t-1} m_{s,t-1} \\ &\stackrel{\text{define}}{=} H_1 p_{s,t} + H_2 m_{s,t} + H_3. \end{aligned} \quad (16)$$

Using (16), the discharge pressure at time t can be expressed linearly. Next, we substitute deviation variables $\Delta p_{s,t} = p_{s,t} - p_{s,t-1}$ and $\Delta m_{s,t} = m_{s,t} - m_{s,t-1}$ from reference point into the variables $p_{s,t}$ and $m_{s,t}$ in (16). The relationship between the deviations Δm_s and Δp_s at suction port, and Δp_d at discharge port of a compressor can be expressed as

$$\Delta p_d = H_1 \Delta p_s + H_2 \Delta m_s. \quad (17)$$

Considering that the mass flow at the suction and discharge of centrifugal compressor should be consistent [36], this implies that the deviation of mass flow at suction port is equal to the discharge port. Therefore, we have

$$\Delta m_d = \Delta m_s. \quad (18)$$

Similarly to the two-port pipeline expression, we combine Eqs. (17) and (18) and transform them into the Laplace domain. The matrix form is written as

$$\begin{bmatrix} \Delta \Pi_d \\ \Delta M_d \end{bmatrix} = \begin{bmatrix} H_1 & H_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta \Pi_s \\ \Delta M_s \end{bmatrix}. \quad (19)$$

Then, the relationships between pressures and flows at the suction and discharge port of compressor are explicitly expressed by using the transfer matrix.

D. General Transfer Function Model for Dynamics in NGS

For the sake of analysis, we assume that the compressor is connected to the upstream port of the pipeline, as shown in Fig. 2. The pipeline is cascaded with the compressor. Thus, the output of the compressor cascade section is the input for the pipeline cascade section. By applying Eqs. (8) and (19) and eliminating the identical physical variables in the discharge

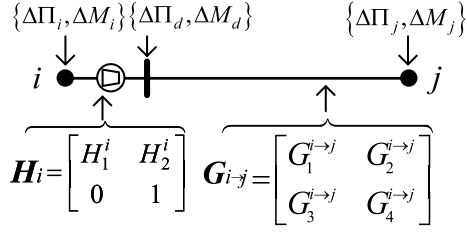


Fig. 2. Diagram of pipeline and compressor cascade.

node of compressor, the cascaded transfer function of the pipeline with compressor is derived as follows

$$\begin{aligned} \begin{bmatrix} \Delta \Pi_j \\ \Delta M_j \end{bmatrix} &= \begin{bmatrix} G_1^{i \rightarrow j} H_1^i & G_1^{i \rightarrow j} H_2^i + G_2^{i \rightarrow j} \\ G_3^{i \rightarrow j} H_1^i & G_3^{i \rightarrow j} H_2^i + G_4^{i \rightarrow j} \end{bmatrix} \begin{bmatrix} \Delta \Pi_i \\ \Delta M_i \end{bmatrix} \\ &= \begin{bmatrix} T_1^{i \rightarrow j} & T_2^{i \rightarrow j} \\ T_3^{i \rightarrow j} & T_4^{i \rightarrow j} \end{bmatrix} \begin{bmatrix} \Delta \Pi_i \\ \Delta M_i \end{bmatrix}. \end{aligned} \quad (20)$$

The new transfer functions $T_{1/2/3/4}^{i \rightarrow j}$ encompass not only the pipeline parameters, but also the compressor characteristic. When there is no compressor on the pipeline, the $H_1 = 1$ and $H_2 = 0$, then, $T_1^{i \rightarrow j} = G_1^{i \rightarrow j}$, $T_2^{i \rightarrow j} = G_2^{i \rightarrow j}$, $T_3^{i \rightarrow j} = G_3^{i \rightarrow j}$ and $T_4^{i \rightarrow j} = G_4^{i \rightarrow j}$, which are consistent with transfer functions in (8). When there is a compressor considering the static model, the $H_1 > 1$ and $H_2 = 0$. Hence, (20) can be used to generically express dynamic models of pipeline with and without a compressor.

When the transfer matrix is extended to a network, we need supplement the law of mass flow conservation at any node i to realize the supply and demand balance:

$$\Delta M_i^s - \Delta M_i^l + \sum_{Bi} M_i - \sum_{Bo} \Delta M_i = 0, \quad (21)$$

where ΔM_i^s , and ΔM_i^l are the gas flow injected or demand at node i ; Bi and Bo are the branch sets connected with node i . $\sum_{Bi} M_i$ and $\sum_{Bo} M_i$ are the total gas flow from the branches input or output node i , respectively.

E. Analytical Expression of Dynamic Response in NGS

In the dynamic analysis, the pressure variations transfer from upstream to downstream of the network, while the flow variations transfer from downstream to upstream. Thus, the states of pressure at the end node and mass flow at the start node are concerned. Based on this, we re-adjust the input and output positions in model (20), given by

$$\begin{bmatrix} \Delta \Pi_j \\ \Delta M_i \end{bmatrix} = \begin{bmatrix} \phi_1^{i \rightarrow j} & \phi_2^{i \rightarrow j} \\ \phi_3^{i \rightarrow j} & \phi_4^{i \rightarrow j} \end{bmatrix} \begin{bmatrix} \Delta \Pi_i \\ \Delta M_j \end{bmatrix}, \quad (22)$$

where $\phi_1^{i \rightarrow j}$, $\phi_2^{i \rightarrow j}$, $\phi_3^{i \rightarrow j}$, and $\phi_4^{i \rightarrow j}$ are the transfer function derived based on $T_{1/2/3/4}^{i \rightarrow j}$, shown as

$$\begin{aligned} \phi_1^{i \rightarrow j} &= T_1^{i \rightarrow j} - \frac{T_2^{i \rightarrow j}}{T_4^{i \rightarrow j}} T_3^{i \rightarrow j}, & \phi_2^{i \rightarrow j} &= \frac{T_2^{i \rightarrow j}}{T_4^{i \rightarrow j}} \\ \phi_3^{i \rightarrow j} &= -\frac{T_3^{i \rightarrow j}}{T_4^{i \rightarrow j}}, & \phi_4^{i \rightarrow j} &= \frac{1}{T_4^{i \rightarrow j}}. \end{aligned} \quad (23)$$

Then, we can deduce that there is a relationship between the pressure fluctuations and the flow fluctuations, both of which correspond to variations in load demand

$$\Delta \Pi_j = \phi_1^{i \rightarrow j} \Delta \Pi_i + \phi_2^{i \rightarrow j} \Delta M_j, \quad (24)$$

$$\Delta M_i = \phi_3^{i \rightarrow j} \Delta \Pi_i + \phi_4^{i \rightarrow j} \Delta M_j. \quad (25)$$

When the start node is a slack node with fixed pressure, the term $\Delta \Pi_i = 0$, and therefore, it can be omitted.

For branches with node i as start node and node j as end node, we extend model (22) and write it in matrix form as

$$\begin{bmatrix} \Delta \Pi_{i+1} \\ \Delta M_i \\ \vdots \\ \Delta \Pi_j \\ \Delta M_{j-1} \end{bmatrix} = \begin{bmatrix} \phi_1^1 & \phi_2^1 & & & \\ \phi_3^1 & \phi_4^1 & & & \\ & & \ddots & & \\ & & & \phi_1^{n_p} & \phi_2^{n_p} \\ & & & \phi_3^{n_p} & \phi_4^{n_p} \end{bmatrix} \begin{bmatrix} \Delta \Pi_i \\ \Delta M_{i+1} \\ \vdots \\ \Delta \Pi_{j-1} \\ \Delta M_j \end{bmatrix}, \quad (26)$$

where n_p is the pipelines from i to j . Taking the terminal node j as an example, by using the backward elimination of the tri-diagonal matrix, we can derive the explicit formula of the relationship between the nodal pressure fluctuations at j and load demand variations at other nodes in the branch:

$$\Delta \Pi_j = \left(\prod_{q \in \Phi_{ji}} \phi_1^q \right) \Delta \Pi_i + \sum_{k \in \Phi_j} \left[\left(\prod_{l \in \Phi_{jk}} \phi_1^l \right) \phi_2^k \Delta M_k \right], \quad (27)$$

where Φ_{ji} is the index set of pipelines on the path from node j to node i . Φ_j is the index set of nodes on the pipelines in Φ_{ji} (excluding nodes i and j). Φ_{jk} is the index set of pipelines on the path from node j to node k . ϕ_2^k is the transfer function of the pipelines with nodes k as the end node. Similarly, when the start node is a slack node, the term $(\prod_{q \in \Phi_{ji}} \phi_1^q) \Delta \Pi_i$ can be omitted. Then, the pressure fluctuation at node j with the demand variations at different nodes can be quantified. Similar analysis applies to other critical nodes. Note that the term $(\prod_{l \in \Phi_{jk}} \phi_1^l) \phi_2^k$ will introduce high order Laplacian “s”, which will make the subsequent analysis difficult. Hence, a model order reduction method is needed to reduce the complexity while preserving the dynamic properties. The reduced method will be discussed in the next section.

The above models offer the explicit expression between the pressure fluctuation of any node and flow demand variation in an analytical form. Once given the physical initial solution, the nodal state can be calculated directly, which avoids the intensive and repetitive numerical computations associated with traditional iterative methods and FDM.

III. REDUCTION METHOD FOR THE GENERAL TRANSFER FUNCTION MODEL

A. Topology Reduction

Based on the characteristics of different components in dynamic analysis, the nodes in NGS can be divided into four types [33]:

- (1) a slack node with fixed-pressure that connects to the gas source and determines the level of pressure in the network;
- (2) flow nodes that connect to the gas storage or traditional gas loads;

(3) intersection nodes that gather pipelines;

(4) critical nodes of interest or importance, such as those connected to GPG and EDC or the terminal of network. Among them, the intersection nodes have no demand variation. The flow nodes compared to GPG nodes, usually have small demand variation and can be considered as flow-fixed nodes. Hence, the types (2) and (3) can be eliminated to reduce the model size for a concise analysis [33]. In this paper, we aggregate multiple pipelines that are clustered by intersection nodes or flow-fixed nodes into an equivalent pipeline, thereby deriving a topological reduction model of relationships between only significant nodes. Taking a gas branch with n_b intersection nodes as an example, the transfer matrix of equivalent pipeline can be obtained through the cascade of identical physical variables of each pipe on the branch, which is shown as

$$\begin{bmatrix} \Delta \Pi_{n_b} \\ \Delta M_{n_b} \end{bmatrix} = \begin{bmatrix} T_1^{n_b-1 \rightarrow n_b} & T_2^{n_b-1 \rightarrow n_b} \\ T_3^{n_b-1 \rightarrow n_b} & T_4^{n_b-1 \rightarrow n_b} \end{bmatrix} \times \dots \times \begin{bmatrix} T_1^{1 \rightarrow 2} & T_2^{1 \rightarrow 2} \\ T_3^{1 \rightarrow 2} & T_4^{1 \rightarrow 2} \end{bmatrix} \times \begin{bmatrix} \Delta \Pi_1 \\ \Delta M_1 \end{bmatrix} = T_e \begin{bmatrix} \Delta \Pi_1 \\ \Delta M_1 \end{bmatrix}, \quad (28)$$

where T_e represents the cascaded transfer matrix of the equivalent pipeline, which is multiplied by the transfer function of pipelines in the path from node n_b to node 1. It can be found that the larger the number of pipelines, the higher order the Laplacian “s” of T_e .

B. Model Order Reduction

The model orders in Eqs. (27) and (28) depend on the network size. For example, the transfer function of a single pipeline is second-order, while an equivalent transfer function of n cascade pipelines is $2n$ -order. To simplify analysis, the high-order elements in the transfer function models are reduced to a low-order model while preserving the main frequency features. On the one hand, the filtering effect of natural gas network makes the low-frequency dynamics play a leading role [37]. On the other hand, the dynamics in NGS are very slow compared to EPS, thus the high-frequency dynamics have little impact on IEGS and can be neglected [25]. To this end, we derived the reduced model via Jordan continued-fraction expansion at low-frequency points [38].

We give a high-order expanding transfer function, i.e.,

$$T_N(s) = \frac{a_{2,z}s^{z-1} + \dots + a_{2,2}s + a_{2,1}}{a_{1,z+1}s^z + \dots + a_{1,2}s + a_{1,1}} \triangleq \frac{A_2(s)}{A_1(s)}, \quad (29)$$

where z is the order of the transfer function. $a_{1,1}, \dots, a_{1,z+1}$ and $a_{2,1}, \dots, a_{2,z}$ are the expansion coefficients of the denominator and numerator polynomials. We rewrite T_N as:

$$T_N(s) = \frac{1}{\frac{A_1(s)}{A_2(s)}} = \frac{1}{(s^2 + w_1^2)T'_N(s) + b_1s + h_1} \quad (30)$$

where w_1 is the first expanding frequency, which is selected to be $w_1 = 0$. This choice is made to ensure an adequate fit for the original characteristics in the low frequency [39]. b_1

and h_1 are the first partial quotient pairs. $T'_N(s)$ is expressed by introducing a new polynomial $A_3(s)$, shown as

$$T'_N(s) = \frac{a_{3,z-1}s^{z-2} + \dots + a_{3,2}s + a_{3,1}}{a_{2,z}s^{z-1} + \dots + a_{2,2}s + a_{2,1}} \triangleq \frac{A_3(s)}{A_2(s)}. \quad (31)$$

To determine $A_3(s)$, we substitute (31) in (30) and expand the coefficients of $A_2(s)$ and $A_1(s)$. Then, the coefficients $a_{3,j}$ of s^{j-1} in $A_3(s)$ can be obtained as follows:

$$a_{3,j} = a_{1,j+2} - h_1 a_{2,j+2} - k_1 a_{2,j+1} - w_1^2 a_{3,j+2}, \quad j = z-1, z-2, \dots, 1, \quad (32)$$

where $a_{i,j} = 0$ for $j \geq z - i + 3$, $i = 1, 2, 3$.

By substituting $s = jw_1$ and $s = -jw_1$ into (30) respectively, we can derive b_1 and h_1 , shown as

$$h_1 = \frac{1}{2} \frac{A_1(s)A_2(-s) + A_1(-s)A_2(s)}{A_2(s)A_2(-s)}, \quad (33)$$

$$b_1 = \frac{1}{2} \frac{A_1(s)A_2(-s) - A_1(-s)A_2(s)}{sA_2(s)A_2(-s)}. \quad (34)$$

Similar to the above steps, the second partial quotient pairs can be derived and shown as follows:

$$h_2 = \frac{1}{2} \frac{A_2(s)A_3(-s) + A_2(-s)A_3(s)}{A_3(s)A_3(-s)}, \quad (35)$$

$$b_2 = \frac{1}{2} \frac{A_2(s)A_3(-s) - A_2(-s)A_3(s)}{sA_3(s)A_3(-s)}. \quad (36)$$

Then, a second-order reduced model can be formed by truncating the Jordan-type continued expansion after the first two partial quotient pairs:

$$T_{R2}(s) = \frac{1}{h_1 + b_1 + \frac{s^2 + w_1^2}{h_2 + b_2s}} = \frac{b_2s + h_2}{(b_1b_2 + 1)s^2 + (h_1b_2 + h_2b_1)s + h_1h_2 + w_1^2}. \quad (37)$$

Similarly, if an m th-order reduced model is desired, we can repeat the above steps to obtain the first m partial quotient pairs $\{h_1, b_1\}, \dots, \{h_m, b_m\}$. Eventually, a general expression of the m th-order reduced model is formed as

$$T_{Rm}(s) = \frac{1}{h_1 + b_1s + \frac{s^2 + w_1^2}{h_2 + b_2s} + \dots + \frac{h_{(m-1)} + b_{(m-1)}s + \frac{s^2 + w_{(m-1)}^2}{h_m + b_ms}}{m = 1, 2, \dots, z-1}. \quad (38)$$

It can be proved that T_{Rm} fits the frequency-response characteristics of the original system at the desired frequencies w_i by using $T_{Rm}(jw_i) = T_N(jw_i)$. The proposed method can obtain a reduced model without complex algebra. Based on the reduced transfer function, we can conveniently transform the L-domain variables $(\Delta \Pi, \Delta M)$ into the time-domain $(\Delta p, \Delta m)$ by using Laplace inverse transform. Then, the analytical expression of pressure and flow fluctuations can be obtained.

Considering that the second-order transfer function includes plenty of features of the dynamic system, we first select the second-order reduction for any higher-order system ($2n$ -order ($n > 1$)). By obtaining the second-order reduction function

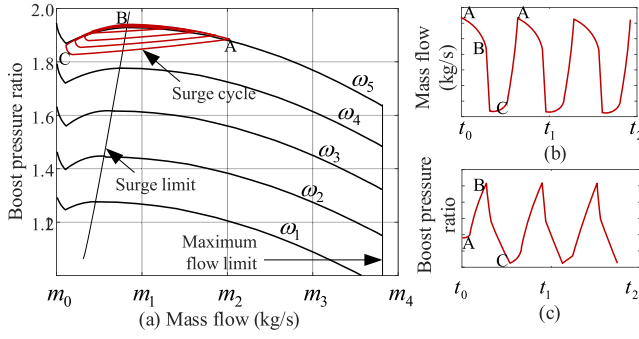


Fig. 3. Performance curve diagram of compressor.

through (33) – (37), the Bode plot and step response curve of the original and the second-order reduced transfer function are compared to observe the error. If the error is larger than the set error threshold, incrementally increase the reduced model's order and repeat the above steps, until the reduced model's response closely matches the original system's response. In general, a second-order or third-order reduction function is sufficient to describe natural gas systems dominated by low frequency characteristics. Finally, the inverse Laplace transformation of the reduced-order function is carried out.

IV. INTERACTION BETWEEN EPS AND NGS

The fluctuations in one system can be transferred to the other system through the coupling units. This section investigates the impact of the coupling units under the role of proposed model.

A. Electric-Driven Compressors

The electric power consumed by compressor P_{EDC} is determined by horsepower [36], given by

$$HP = 3.0177 \frac{z_G T_{GC} p_{0G} m_s}{E_c \eta_c T_{0G} \rho_G \kappa - 1} \left[\left(\frac{p_d}{p_s} \right)^{\frac{\kappa-1}{\kappa}} - 1 \right], \quad (39)$$

$$P_{EDC} = (745.7 \times 10^{-6}) HP, \quad (40)$$

where E_c and η_c are parasitic efficiency and compressor efficiency. z_G is gas compressibility factor. p_{0G} and T_{0G} are standard pressure and temperature. m_s is the mass flow through the compressor. HP is horsepower.

It can be observed that the compressor power consumption is directly affected by the mass flow. For a centrifugal compressor, the operation envelope is limited by a minimum flow and a maximum flow. The operating interval diagram is summarized in Fig. 3. On the one hand, crossing the minimum flow limit will cause periodic large amplitude flow oscillations in the compressor [36], as shown the red cycle in Fig. 3 (a). This phenomenon is called surge, which occurs when the mass flow reduce to the unstable region (left zone of the surge line), resulting pressure and flow changes as shown in Fig. 3 (b) and (c), respectively. We could found deep fluctuations and oscillations in pressure and flow when surge occurs. This phenomenon will directly lead to the damage of compressor and pipeline, while the fluctuation is transmitted

to the power grid to affect energy supply security. On the other hand, crossing the maximum flow limit will lead to choke. This phenomenon affects gas delivery reliability and leads to pressure loss cannot be compensated. Therefore, the flow needs to be strictly limited to ensure the compressor in a stability operation range, shown as

$$m_s^{\min} \leq m_s \leq m_s^{\max}. \quad (41)$$

The minimum flow requires special attention that is defined by the surge point mass flow and an aerodynamic stability limit of the compressor [35], calculated by

$$m_s^{\min} = \frac{c_3 c_4}{c_3 c_4^2 + \mu_G} w. \quad (42)$$

The maximum flow of compressor should not only respect its own limits but also respect the pipeline limits. This ensures that the gas is delivered within normal pressure range.

Correspondingly, the electric power consumed by the compressor will have a stability operation range:

$$P_{EDC}(m_s^{\min}) \leq P_{EDC} \leq P_{EDC}(m_s^{\max}), \quad (43)$$

where $P_{EDC}(m_s^{\min})$ and $P_{EDC}(m_s^{\max})$ respective are the minimum and maximum power consumed by EDC.

B. Gas-Fired Power Generations

As critical gas load, the operation of GPG will be affected by gas pressure fluctuations, which will affect the power supply security. This section investigates the effects of the proposed model on the power output and ramp rate of GPG.

The coupling between the natural gas and electricity brought by GPG is modeled based on the heat rate curve [10]:

$$HR = \alpha_G (P_{GPG})^2 + \beta_G P_{GPG} + \gamma_G, \quad (44)$$

$$m_{GPG} = \frac{\rho_G HR}{Gh\nu}, \quad (45)$$

where α_G , β_G and γ_G are heat rate coefficients. ρ_G and $Gh\nu$ are the density and gross heating value of natural gas, respectively. m_{GPG} is the gas flow consumed by GPG. Since the parameters are all positive constant, m_{GPG} increases monotonically as P_{GPG} . Correspondingly, the power output of P_{GPG} is limited by the operating range of m_{GPG} . If there is an EDC in the pipeline where GPG is located, the output range of P_{GPG} will not only be affected by the flow limitations of the pipeline ($m_{pipe}^{\max}$), but also by the flow limits of EDC:

$$P_{GPG}(m_s^{\min}) \leq P_{GPG} \leq P_{GPG}(\min(m_s^{\max}, m_{pipe}^{\max})) \quad (46)$$

On the other hand, the gas-fired turbines must operate within specified pressure change rate thresholds. Failure to do so can force them offline, jeopardizing the safety of the power system. Therefore, the operation of GPG should comply with the limit of gas pressure change rate to ensure the safety of both power system and gas system.

$$r_p = \frac{\Delta p(m_{GPG} - m_{GPG,0})}{D_p} \leq r_{p,max}, \quad (47)$$

where D_p is the duration time of the pressure ramp. $m_{GPG,0}$ is the gas flow consumed by GPG at initial time. $r_{p,max}$ is the upper limit of pressure change rate of GPG.

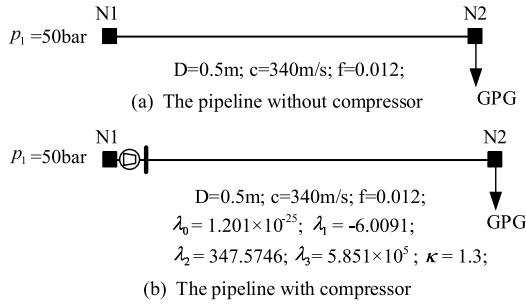


Fig. 4. Diagram in Case I.

V. CASE STUDY

Two cases are investigated to verify the effectiveness of the proposed model. Case I is a single pipeline with one EDC and one GPG, which aims to verify the accuracy of the proposed dynamic model and investigate the impact of the EDCs. Case II is an integrated electricity and gas system, composed of a 14-bus EPS and a 20-node Belgium NGS, which aims to verify the effectiveness of proposed reduced model and investigate the dynamics impact of NGS and EDCs on EPS. All the simulations are coded by MATLAB R2020a on a PC with CPU i5-1135G7, 16G RAM.

A. Case I

The structures and parameters of a single pipeline are shown in Fig. 4. The parameters of compressor are also shown in Fig. 4, and the desired rotational velocity is 600 rpm/s. The pressure at start end N1 remains at $p_1=50$ bar. The terminal end N2 is linked to a GPG unit, and the maximum power output limit is 385MW. The heat rate coefficients of GPG are $\alpha_G = 0.004$, $\beta_G = 13.51$ and $\gamma_G = 176.95$. The mass flow m_2 at N2 changes every hour, as shown in the dashed line in Figs. 5(b), 5(d), and 5(f). The pipelines with compressor and without compressor are both analyzed. The results under different models and different pipeline length are displayed and compared, shown as below.

1) *Pipeline Dynamic Model Validation*: Four models are simulated including the original PDE model (M1) [14]; the FDM-based with discrete criteria model (M2) [23]; the FDT-based with first-order transfer matrix model (M3) [25]; and the proposed second transfer matrix model (M4). The M1 is solved with precise difference steps ($\Delta t_g = 10s$, $\Delta x_g = 500m$). The large step sizes ($\Delta t_g = 600s$, $\Delta x_g = 25000m$) based on discrete criterion are used for M2. The initial transfer function expressions of the pipeline in M3 and M4 are summarized in column 3 of Table I. 7h simulations in different pipeline length of Fig. 4 (a) are conducted to compare the pressure at the downstream port p_2 and the flow at the upstream port m_1 , shown in Fig. 5. It can be seen that all the models are capable of tracking the gas dynamics, with longer pipelines exhibiting slower dynamic changes. The results comparison of several models are summarized in Table II. We can see that FDT-based models (M3, M4) have the lowest equation dimensions and smaller computational time compared to the FDM-based models (M1, M2) during the 7h simulations. In addition, taking the results of M1 as the benchmark, the

TABLE I
THE TRANSFER FUNCTION EXPRESSIONS OF PIPELINE IN CASE I

Transfer function	Different length	pipeline without compressor	pipeline with compressor
$M_3 : \phi_2$	25km	$\frac{-26.57}{0.28s+0.0024}$	$\frac{-26.57}{0.28s+0.0024}$
	50km	$\frac{-15.57}{0.33s+0.00068}$	$\frac{-15.57}{0.33s+0.00068}$
	75km	$\frac{-5.655}{0.18s+0.00016}$	$\frac{-5.655}{0.18s+0.00016}$
$M_3 : \phi_4$	25km	$\frac{-0.28s+0.0024}{0.28s+0.0024}$	$\frac{-0.28s+0.0024}{0.28s+0.0024}$
	50km	$\frac{-0.33s+0.00068}{0.33s+0.00068}$	$\frac{-0.33s+0.00068}{0.33s+0.00068}$
	75km	$\frac{-0.18s+0.00016}{0.18s+0.00016}$	$\frac{-0.18s+0.00016}{0.18s+0.00016}$
$M_4 : \phi_2$	25km	$\frac{-938.32s-79.59}{9.98s^2+0.86s+0.0074}$	$\frac{-0.0054s^2-939.67s-79.71}{9.98s^2+0.86s+0.0073}$
	50km	$\frac{-178.93s-15.54}{3.80s^2+0.33s+0.00068}$	$\frac{-0.0020s^2-179.12s-15.57}{3.80s^2+0.33s+0.00068}$
	75km	$\frac{-63.07s-5.66}{2.01s^2+0.18s+0.00016}$	$\frac{-0.0011s^2-63.07s-5.66}{2.01s^2+0.18s+0.00016}$
$M_4 : \phi_4$	25km	$\frac{-9.98s^2-0.86s+0.0074}{9.98s^2+0.86s+0.0074}$	$\frac{-8.12s^2-0.69s+0.0059}{8.12s^2+0.69s+0.0059}$
	50km	$\frac{-3.80s^2-0.33s+0.00068}{3.80s^2+0.33s+0.00068}$	$\frac{-2.30s^2-0.20s+0.00042}{2.30s^2+0.20s+0.00042}$
	75km	$\frac{-2.01s^2-0.18s+0.00016}{2.01s^2+0.18s+0.00016}$	$\frac{-9.81s^2-0.88s+0.00076}{9.81s^2+0.88s+0.00076}$

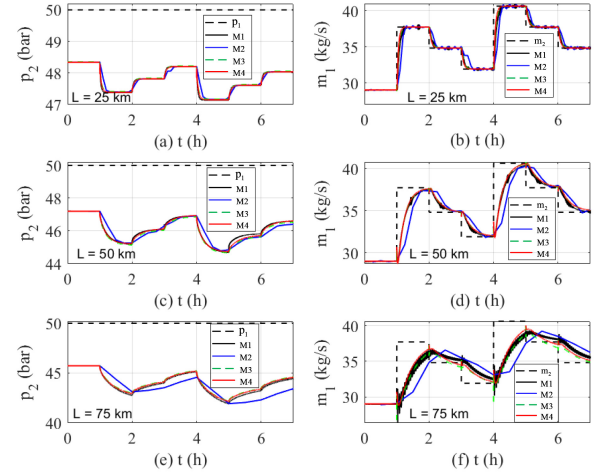


Fig. 5. The result comparison of gas flow dynamics.

TABLE II
RESULTS COMPARISON IN CASE I

Model	Equation Dimensions			Computational time(s)		Maximum relative errors(%)	
	25km	50km	75km	25km	50km	regarding p_2	regarding m_1
M1	2250 × 100	2250 × 100	2250 × 100	65.75	285.63	—	—
M2	42 × 2	42 × 4	42 × 6	1.10	4.67	2.51	7.84
M3	7 × 2	7 × 2	7 × 2	1.37	1.52	0.61	8.94
M4	7 × 2	7 × 2	7 × 2	1.94	2.74	0.52	3.91

maximum relative errors regarding p_2 and m_1 are given. We can see that the M4 has the smaller errors compared to M3. The accuracy improvement of second transfer function model is understandable because it accurately describes the temporal distribution of dynamic gas flow without time discretization, and keeping the inertia term in model.

2) *Compressor Dynamic Model Validation*: The initial transfer function expressions of the pipelines with compressors in M3 and M4 are summarized in column 4 of Table I. It can be found that the transfer function obtained by M3 considering static compressor model is the same as the pipeline transfer function in column 3, which cannot reflect the dynamic change of compressor and its influence on the pipeline state variables. To verify the accuracy of the proposed two-port compressor model in M4, 7h simulations are conducted. The suction pressure and mass flow are consistent with Fig. 5. The results are compared with the original non-linear model, as shown in

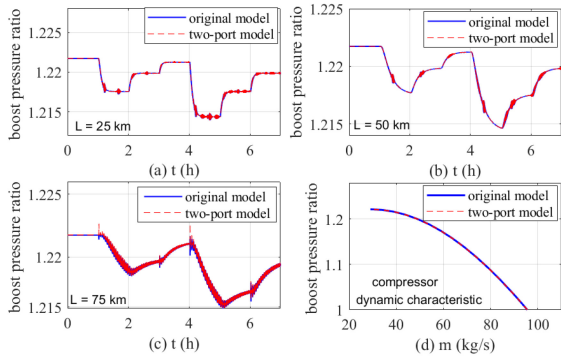


Fig. 6. The result comparison of compressor dynamics.

Fig. 6. Figs. 6(a), 6(b), and 6(c) give the boost pressure ratio change over time for the length of 25km, 50km, and 75km, respectively. It can be seen that the results in the two-port model can track the original model very well. The maximum relative error is 0.24%. The variation range of boost ratio are [1.214, 1.222], [1.215, 1.222], and [1.216, 1.222] for the pipeline with length of 25km, 50km, and 75km, respectively. Fig. 6 (d) simulates the dynamic change of the compressor over mass flow. The greater the mass flow through compressor, the smaller the boost ratio. When the mass flow increases to 95 kg/s, the boost ratio drops to 1, which will lose its boost function.

3) *The Impact of EDC Dynamic:* To investigate the impact of the compressor dynamic on the NGS pressure, the results of the traditional static compressor model and proposed dynamic two-port model are compared. For the former, the compressor boost ratio is fixed at the initial value 1.222 and does not change with the flow status. Thus, the compressor's discharge pressure p_d is always equal to 61.10 bar, as illustrated by the blue line in Figs. 7(a), 7(c) and 7(e). For the latter, the discharge pressures p_d are shown by the red line, which changes with the boost ratio in Fig. 6. Under this impact, the outlet pressures p_2 is decreased. As shown in Figs. 7(b), 7(d) and 7(f), compared with the case without two-port model, the outlet pressure with two-port model drop by up to 79kPa.

Furthermore, we observe the impact of compressor dynamic on the power grid. The surge point mass flow is 28.92kg/s, calculated based on (42). Considering the flow limit within [28.92kg/s, 95kg/s], the electric power output range of GPG could be calculated to be [102.17MW, 341.24MW]. It can be seen that the compressor dynamic introduces a minimum output limit. Compared to the rated output of the GPG, the actual maximum output capacity decreased by 11.37%.

B. Case II

An integrated electricity and natural gas system is adopted to illustrate the effectiveness of the proposed reduced model, and the interaction between EPS and NGS. The structure of this IEGS as well as the initial operation state are shown in Fig. 8 (a), where node 1 (N1) is in pressure control mode, and the pressure remains at 56bar. Two GPGs are installed in N12 and N16, and supply electric power for E1 and E2. Two EDCs are installed in NGS, powered by E4 and E5, respectively.

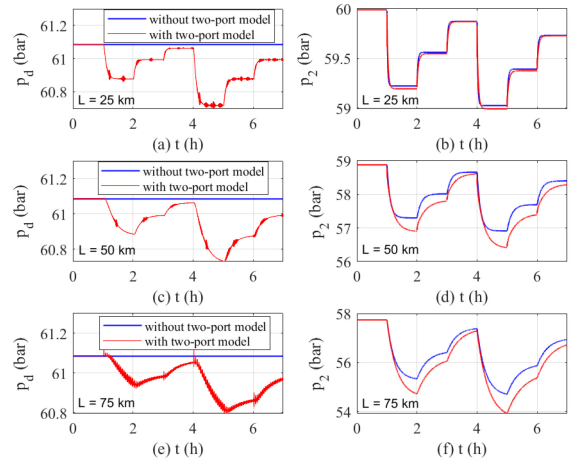


Fig. 7. The impact of the compressor dynamic on the port pressure.

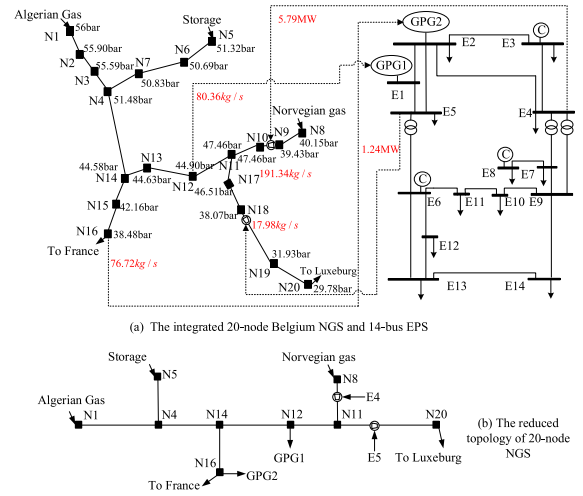


Fig. 8. Diagram in Case II.

1) *Accuracy of Reduced Model:* Firstly, to simplify the network structure, we retain the key coupling nodes N12 and N16 along with terminal nodes. Subsequently, other intersection nodes and cascade pipelines are aggregated into an equivalent pipeline. The reduced structure is shown in Fig. 8 (b). To verify the performance of the reduced structure, we use the pressure response to flow changes at the ends of equivalent branches as examples. Details regarding the original and reduced model of each equivalent branch, as well as their response computational time are shown in Table III. The step responses curves of pressure are illustrated in Fig. 9. It can be found that the responses from the reduced-order transfer function (represented by blue dashed line) are almost consistent with those from the full-order model (represented by red line). Note that the response time of pipelines 1-4 and 12-14 are several hundred seconds, which are faster than other pipelines with thousands of seconds. Therefore, pipelines 1-4 and 12-14 are chose to be reduced to 3-order to remain more characteristic of low frequency, while reducing to 2-order is enough to simulate full-order performances for other pipelines. In addition, we can notice that the pipeline 11-20 has a very large pressure drop. 1kg/s flow change at terminal

TABLE III
THE RESULTS COMPARISON BETWEEN ORIGINAL AND
REDUCED MODEL IN CASE II

Equivalent Branches	Original Model		Reduced Model		
	Order	Time(s)	Order	expression of ϕ_2	Time(s)
1-4	6	1.09	3	$-20.03s^2 - 0.19s - 0.051$	0.14
4-5	6	0.59	2	$0.46s^3 + 0.012s^2 + 0.0023s + 1.17e-5$	0.15
8-11	6	0.54	2	$-21.11s - 0.12$	0.13
12-14	4	0.26	3	$0.19s^2 + 0.010s + 1.20e-5$	0.20
14-16	4	0.21	2	$-14.66s - 0.10$	0.16
11-20	8	3.23	2	$0.044s^2 + 0.0025s + 2.62e-6$	0.13
				$-198.70s^2 - 1.74s - 0.47$	
				$0.60s^3 + 0.067s^2 + 0.013s + 3.39e-4$	
				$-17.11s - 0.26$	
				$0.011s^2 + 0.018s + 4.35e-5$	
				$-141.0s - 0.069$	
				$0.62s^2 + 0.0021s + 6.32e-8$	

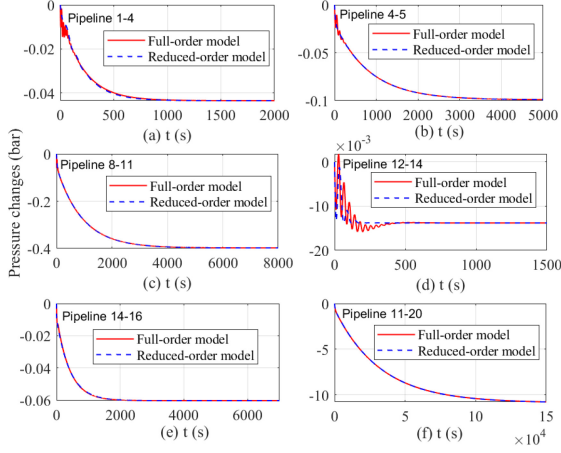


Fig. 9. Step (1kg/s) response of pressure at the end of each equivalent branch.

end N20 causes a 10 bar pressure drop, which should be focused on.

2) *Analytical Expression of Pressure at Keys Nodes:* Here, we concentrate on the pressure responses (p_{12} and p_{16}) of coupling nodes N12 and N16 in relation to the power changes of GPG1 and GPG2. It is assumed that the two GPGs need to balance the power fluctuations within one minute. When GPG2 at E2 undergoes a step change with a 10MW increase. Consequently, based on the power flow balance, GPG1 at E1 experiences a corresponding step change with a 10.08MW decrease. Then, we can derive the pressure responses (Δp_{12} and Δp_{16}) based on the 10MW and 10.08 MW equivalent gas flow. Next, by using the inverse Laplace transformation, the equivalent model of pressure change (p_{12} and p_{16}) in the time domain can be analytically given by:

$$p_{12}(t) = 0.43e^{-1.3 \times 10^{-4}t} + 0.32e^{-1.2 \times 10^{-3}t} + 0.054e^{-0.15t} + 0.62e^{-1.6 \times 10^{-4}t} - 0.084e^{-0.18t} - 1.38e^{-2.0 \times 10^{-4}t} - 0.098e^{-2.2 \times 10^{-3}t} + 45.038(\text{bar}), \quad (48)$$

$$p_{16}(t) = 0.30e^{-9.9 \times 10^{-4}t} + 0.48e^{-1.0 \times 10^{-3}t} - 0.00029e^{-0.65t} + 0.0016e^{-0.18t} - 1.99e^{-2.0 \times 10^{-4}t} + 0.32e^{-0.81t} + 2.094e^{-4.3 \times 10^{-4}t} + 37.207(\text{bar}). \quad (49)$$

The response curves are displayed in the red lines in Figs. 10 (a) and 10(b). To verify the accuracy of the analytical model, the traditional PDE model (M1) and steady-state model (M5) are adopted to compare. The results are shown as blue and dark lines, respectively. According to the figures, several models have changed to the same steady-state value after the power demand change of GPG2. The results from M4 closely

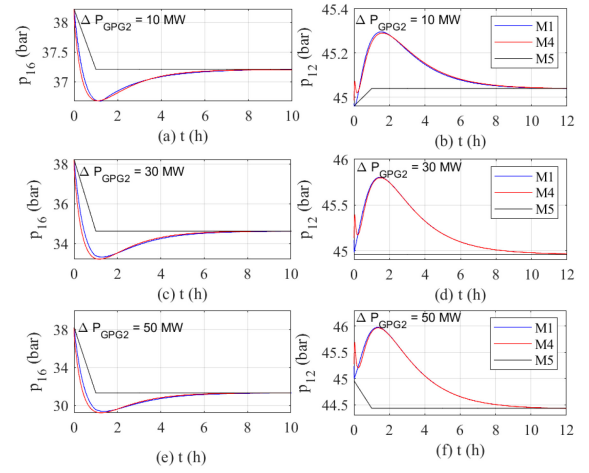


Fig. 10. Pressure changes of coupling nodes N12 and N16 in relation to the power changes of GPG1 and GPG2.

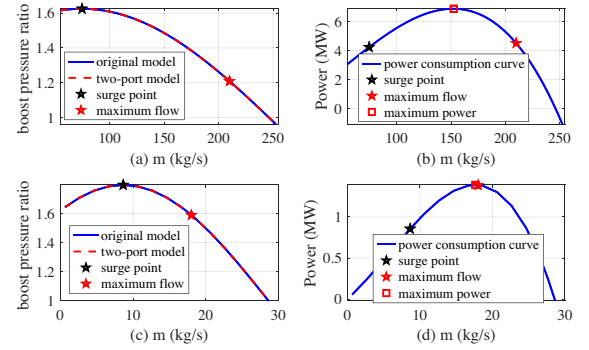


Fig. 11. The dynamic curves of boost pressure ratio of: (a) EDC1 (c) EDC2; and power consumption of: (b) EDC1 (d) EDC2.

track those from M1, where the pressure at N16 decreases due to its increased power demand and then increases due to the pressure change of N12. However, the M5 ignores the dynamic response of pressure change and only reflects the static value. Similar simulations are conducted for a 30MW and 50 MW increase step change of GPG2 at E2. The results are shown in Figs. 10(c)-10(f). As the system dynamics change until it enters a steady state, the maximum error between M1 and M4 decreases from 1.36% to 0.028% for p_{16} , and from 0.85% to 0.00066% for p_{12} .

The proposed reduced model efficiently derives an explicit expression for time-varying pressure variations. This approach not only circumvents the need for repeated iterative calculations of typical traditional models, but it also does not require revealing detailed network parameters to EPS.

3) *Operation Limits of EDCs:* Figs. 11 shows the dynamic curves of pressure boost ratio and consumed electrical power as the flow changes for EDC1 and EDC2. The dynamic pressure boost ratio calculated by the proposed two-port model can completely track the original non-linear model, which has satisfactory accuracy. The surge points are calculated based on the surge mass flow (42), which are respective 74.57 and 8.65 kg/s for EDC1 and EDC2 (indicated by black stars). The upper flow limit are 210 and 18.03 kg/s respectively (shown

in the red stars). Based on the above two flow limitations, we can determine that the actual safe operating range of boost ratio and power consumption are 1.63–1.21 and 4.23–6.87 MW for EDC1, and 1.80–1.59 and 0.86–1.39 MW for EDC2. The operating ranges of the two EDCs have been reduced by 61.57% and 61.87%, respectively.

4) *Operation Limits of GPG*: By giving the limit of the rate of pressure change, the ramp rate of GPG by using the proposed model could be obtained. Assuming the upper limit of pressure change rate drops from 80 kPa/s to 60 kPa/s, the generator ramp rates will change from 24.9 MW/min and 18.6 MW/min, with a 33.8% reduction.

The results demonstrate that limitations from NGS significantly affect the operation of power system. The obtained results could provide detailed constraints for EPS analysis.

VI. CONCLUSION

This paper proposes a novel model for dynamic analysis of IEGS. The model is based on transfer function, describing the dynamic characteristics of NGS with EDC and gives the explicit expressions of state fluctuations. Then, a reduced-order method is proposed to reduce the computational burden, so as to improve the efficiency of dynamic analysis. Numerical simulations are conducted in two test systems, and the following conclusions are observed:

1) The proposed second-order general transfer function model can track the gas dynamic with low equation dimensions and high precision results.

2) The dynamic characteristics of EDC have significant impact on both NGS (such as the gas pressure drop, mass flow operation range) and EPS (such as the output capacity of GPG and consumed electric power of EDC), which help provide more detailed operation constraints for IEGS.

3) The reduced model facilitates deriving analytical expressions for nodal pressure and flow fluctuations, and the operating information of critical coupling nodes, such as power output and ramp rate of GPG, can be obtained without a lot of repeated numerical calculations.

In this way, the power system operators can greatly reduce the computational burden by only using a concise gas equation. Future work will be dedicated to the optimization techniques and optimal control method based on our proposed model to improve system operation potential. Meanwhile, the impact of the surge oscillations process on the power grid supply will be studied.

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