

Damping technique empowered robust energy flow calculation for integrated energy systems

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ABSTRACT

Steady-state energy flow calculation (EFC) is the cornerstone of the long-term operation assessment of integrated energy systems (IES). The conventional EFC methods mainly focus on calculation speed and efficiency. However, the robustness is often ignored, which may result in a failure in finding the solutions under ill-conditioned and unsolvable situations. To address this issue, this paper proposes a robust EFC method for integrated electricity-gas-heat system. First, a novel robust EFC method based on Levenberg–Marquardt (L–M) method is proposed for a comprehensive formulation of the energy flow model combining the electricity, natural gas, district heating system and detailed interdependence. Further, a consecutive adaptive damping factor updating approach is proposed. It enables a secure search step to avoid the flutter in iteration process, thus enhancing computational efficiency and convergence for EFC. In addition, the influence of the direction and the size of search step are jointly considered. It is helpful to guarantee solutions for both ill-conditioned and unsolvable situations. Numerical simulation results demonstrate the strong robustness, convergence and universality of the proposed EFC method that works under diversified operation modes, e.g., well-conditioned mode, inappropriate initial values, heavy load mode, and ill-conditioned operation mode.

1. Introduction

The depletion of fossil energy and the high penetration of renewable have brought significant attention to the integrated energy system (IES) [1,2]. IESs offer numerous advantages due to the power adjustment capabilities of energy conversion units, such as gas fired combined heat and power (CHP), gas-fired power generator (GPG), power-to-gas (P2G) units and electric boilers (EB). As a result, the electric power system (EPS) is tightly coupled with the natural gas system (NGS) and the district heating system (DHS) [3–5]. To analyze the inter-dependency and interaction among EPS, NGS, and DHS, the steady-state energy flow calculation (EFC) is widely studied, particularly for long-term research such as planning, reliability assessment and so on [6]. It can realize fast assessment and snapshot of the system state at a single time profile [7,8].

With the increasing complexity of IES, the steady-state EFC can be classified into three categories [9–11]: (i) well-conditioned case, where traditional EFC method can obtain accurate solutions efficiently; (ii) ill-conditioned but solvable case, where traditional EFC method may not obtain solutions (for instance, Newton's method cannot converge if the initial values are far from the real operation point); (iii) unsolvable

case, where there are no feasible solutions. An ideal steady-state EFC method should not only have good performance under well-conditioned cases, but also provide desirable solutions under ill-conditioned and unsolvable cases.

Several studies have proposed to solve EFC efficiently. Newton–Raphson (NR) method is the most commonly used method for steady-state EFC. By using NR method, A. Martinez-Mares et al. studied a general steady-state energy flow analysis approach for the electric-gas IES [8]. Liu et al. studied the integrated and decomposed analysis of energy flow for electric-heat IES, respectively [12]. The results show that integrated analysis can take into account detailed coupling relationships by using Jacobian matrix, and requires fewer iterations than decomposed analysis. Literature [13] proposed an integrated energy flow analysis framework for combined electric, gas and heat networks on transmission level and offered the expansion Jacobin matrix. Literature [14] studied the bi-directional energy flow calculation with GPG and P2G units. Literature [15] integrated the multi-type gas sources in bi-directional energy flow analysis to obtain more accurate operation results. Those studies are based on NR methods that feature fast convergence. However, the NR methods are sensitive to initial

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values, and only obtain convergent results for most well-conditioned systems with appropriate initial values. As the increasing system scale and complexity of IES, the inappropriate initial value problems are more likely to occur.

To address this issue, several researches have been done on reliable and efficient calculation of energy flow. Literature [11] proposed an initial guesses strategy for EFC. However, there are some limitations of this strategy. Literature [16] presented a new decomposing strategy for EFC, in which holomorphic embedding (HE) method and graph method were used to replace NR method in EPS and DHS, respectively. The HE method with less-computational burden is suitable for large-scale IES. However, there still exist ill-conditioned problem in NGS caused by inappropriate initial values of gas pressure. In [17], a fast decoupled method for EFC was proposed, which replaced the Jacobian matrix based on NR method with a diagonal and constant Jacobian matrix. The method can effectively improve the calculation efficiency, but is not suitable for complex networks. In [18], a fixed-point based distributed method with the consideration of information exchange structures was proposed for EFC, which can converge to an accurate solution with less computation time. In [19], an approximate linear gas model was proposed for EFC, in which the initial value can be arbitrary with reduced computational burdens. However, the district heating network cannot be included. Literature [20] proposed a partial decoupling method for fast and reliable EFC for large-scale IES, but without considering gas systems.

Most of the research mentioned above are focused on achieving fast calculation for energy flow. Some of them employ NR method to solve a set of detail nonlinear equations to realize fast quadratic convergence [8,12–15], but the method is limited to system conditions. The others aim to improve the robustness of EFC by using non-iterative method with a set of simplified model [16–20]. However, the accuracy of EFC result is decreased. Moreover, the above methods are only applied to electric-heat IES, electric-gas IES, or electric-gas-heat IES with simplified coupling units. In summary, there is still a lack of robust EFC method that is suitable for integrated electric-gas-heat systems in multi-categories cases: (i) well-conditioned cases; (ii) ill-conditioned but solvable cases; (iii) unsolvable cases.

To tackle this challenge, the damping technique is usually introduced to improve the convergence performance of algorithms. One representative approach is the Levenberg–Marquardt (L–M) method, which introduces a damping factor to prevent the correction variable from becoming too large when the Jacobian matrix is nearly singular [21–23]. Therefore, it is helpful in solving ill-conditioned cases. In addition, it can achieve the least square solution, which serves as an approximate solution, for unsolvable cases. Literatures [24–26] had studied the robust power flow based on L–M method. However, these methods use fixed adjustment factor formulations in iteration process, making it unable to adjust the step size and direction based on the quality of the approximation solutions. Literatures [9,10,27,28] proposed the adaptive damping factors or scalar multiplier at each iteration to help improve convergence. Nevertheless, the factors with the non-consecutive updating strategy that can lead to oscillations or flutter in the convergence process, resulting in reduced convergence efficiency. This limits its application to IES with several complicated and sensitive variables, which calls for new effective solutions.

To ensure robust and efficient energy flow calculation for integrated electric-gas-heat system, this paper proposes a novel method using a damping technique. The main contributions of this paper are as follows.

1. A novel energy flow calculation method is proposed for IES consisting of electricity, natural gas, heating systems and detailed bi-directional interdependence. The method uses an adaptive damping factor, making it a globally convergent L–M method with universality in different categories cases, i.e. well-conditioned, ill-conditioned and unsolvable cases.

2. A consecutive damping factor updating strategy is proposed. It can adaptively update the damping factor to provide a secure search step instead of using a fixed factor as in traditional method. With this effect, the flutter in iteration process can be avoided, thus improving the convergence efficiency.
3. The direction and size of search step are both updated with the damping factor, which guarantee the computational convergence. With this component, the singularity and ill-condition of coefficient matrix can be avoided to further enhance the robustness.

This paper is organized as follows. The integrated energy system modeling of EPS, NGS, DHS and interdependencies are given in Section 2. The problem formulation and the robust EFC method are described in Section 3. Finally, several case studies are carried out on two typical systems in Section 4. Conclusions are given in Section 5.

2. Integrated energy system modeling

2.1. Electric power system (EPS)

We consider the calculation of power flow in AC network [29]. The active P_{ij} and reactive power Q_{ij} of a transmission line between bus i and j is given by:

$$P_{ij} = |V_i||V_j|(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}), \quad (1)$$

$$Q_{ij} = |V_i||V_j|(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}), \quad (2)$$

where $|V_i|$ and $|V_j|$ are voltage magnitudes of bus i and j ; θ_{ij} is the difference between voltage angles θ_i and θ_j ; G_{ij} and B_{ij} are the electricity conductance and the susceptance of transmission line ij , respectively.

The steady-state power flow equilibrium point is defined by the Kirchhoff's law. Specifically, the power injected into each node is equal to the power consumed at this node. To overcome the limitation of only single slack node to compensate the entire transmission losses and power consumption of the EPS, the distributed slack nodes (DSB) model is analyzed in this paper, which has been developed by [8,13]. In DSB model, the active power and losses can be balanced by multiple generators based on their assigned participating factors. In addition, the adjustment of active power generations as a function of gas supply and moto-compressor consumptions as a function of electric supply are computed during the energy flow solution in a closed loop, which more realistically represents the interdependency between both infrastructures. The active power output of regulating generators at each distributed slack node is defined as

$$P_i^{Gen} = P_i^{Gen,0} + k_i \Delta P_b \quad \forall i = 1, 2, \dots, N_{ds}, \quad (3)$$

$$\sum_{i \in N_{ds}} k_i = 1, \quad (4)$$

where $P_i^{Gen,0}$ is the set-point of active power of each participating unit. k_i is the participation factor. ΔP_b is the unknown active power to be balanced. N_{ds} is the total number of distributed slack nodes. In this model, besides the traditional node types PV and PQ , two new node types are defined as $\tilde{P}V$ and $\tilde{P}Q$ with variable active power generation [8]. The active power output limits are related to the operational constraint of generations, the amount of natural gas that can be extracted from the gas network, and the heat power output limits. If any regulating generator reaches one of its active power generation limits, its active power output is fixed at the limit for the remaining of the power flow solution process. The node type will be switched from $\tilde{P}V$ and $\tilde{P}Q$ to PV and PQ , and then the active power balance of the system is provided by the rest of regulating generators.

On the other hand, the reactive power output limits should be also considered. The reactive power output of $\tilde{P}V$ or PV node is calculated during each iterative to judge whether it exceeds the limits. If there is

not required reactive power support to ensure the voltage magnitude at the specific value, the node type will be switched from $\tilde{P}V$ or PV to $\tilde{P}Q$ or PQ . Then the reactive power is fixed at the limit value and the voltage magnitude is unknown variable [8].

Then, the nodal active and reactive power balance equations for the EPS are given by:

$$\Delta P_i = P_i^{Gen} - P_i^{EL} - \sum_{j=1}^{N_e} P_{ij} \quad \forall i = 1, 2, \dots, N_e, \quad (5)$$

$$\Delta Q_i = Q_i^{Gen} - Q_i^{EL} - \sum_{j=1}^{N_e} Q_{ij} \quad \forall i = 1, 2, \dots, N_{pq} + N_{\bar{p}q}, \quad (6)$$

where P_i^{Gen} is the active power of generators injected into node i , including the active power output of coupling units GPG and CHP. P_i^{EL} is the consumed active power at node i . In addition to the traditional electrical load, the consumed power includes coupling units, i.e. EB, P2G, moto-compressor. Q_i^{Gen} and Q_i^{EL} are generated and consumed reactive power at node i , respectively; N_e is the total buses number of EPS; $N_{pq} + N_{\bar{p}q}$ is the total buses number without voltage-controlled.

Assume that the first bus is chosen to be reference for the voltage angle. Then, the unknown vector of state variables is $[X_e] = [\Delta P_b, \theta, V]$, and the nodal active power balance equation (5) is applied to all nodes of electricity network.

2.2. Natural gas system (NGS)

The operation of natural gas system meets the mass conservation law and the Bernoulli equation. Following [13,30–33] and assuming an isothermal gas flow in a long horizontal pipeline, the steady state model of natural gas system is specified by equations related to gas flow, pressure and pipeline parameters. The gas flow f_{ij} through a pipeline connected between node i and j is given by:

$$f_{ij} = C_i \text{sign}_p(\pi_i, \pi_j) \sqrt{\frac{\text{sign}_p(\pi_i, \pi_j) \cdot (\pi_i^2 - \pi_j^2) D_g^5}{\chi L_g T_g Z \sigma}}, \quad (7)$$

where C_i is pipeline coefficient. π_i and π_j are pressures at node i and j , respectively. $\text{sign}_p(\pi_i, \pi_j)$ is a sign function of pressure where $\text{sign}_p(\pi_i, \pi_j) = +1$ if $(\pi_i^2 - \pi_j^2) \geq 0$, otherwise $\text{sign}_p(\pi_i, \pi_j) = -1$. χ is the friction factor. L_g and D_g are the length and diameter of pipeline, respectively. T_g is the gas temperature. Z is the gas compressibility factor. σ is the gas density relative to air. To maintain the pressure level, compressors in gas network are needed to installed. For a compressor connected between node i and j , its energy consumption is:

$$H P_i^{com} = \frac{\lambda_G}{\lambda_G - 1} E_i^{com} f_{ij} \left(\left(\frac{\pi_j}{\pi_i} \right)^{\frac{\lambda_G - 1}{\lambda_G}} - 1 \right), \quad (8)$$

where E_i^{com} is the parameters related to the compressor efficiency, gas compressibility factor and gas temperature at the compressor node [8]. λ_G is the specific heat ratio of gas. If the compressor is driven by natural gas turbine, the consumed gas which is extracted from NGS is given by:

$$f_i^{com} = \alpha_i^C + \beta_i^C H P_i^{com} + \gamma_i^C (H P_i^{com})^2, \quad (9)$$

where α_i^C , β_i^C , and γ_i^C are consumption coefficients of the compressor.

The steady-state equilibrium point for NGS is specified by the nodal gas flow balance equation, i.e.,

$$\Delta f_i = f_i^{GS} + f_i^{P2G} - f_i^{GPG} - f_i^{CHP} - f_i^{GB} - f_i^{Com} - f_i^{GL} - \sum_{j=1}^{N_g} f_{ij} \quad \forall i = 1, 2, \dots, (N_g - 1), \quad (10)$$

where f_i^{GS} and f_i^{P2G} are the injected gas flow of sources and P2G units into node i ; f_i^{GPG} , f_i^{CHP} , f_i^{GB} , f_i^{Com} and f_i^{GL} are the consumed gas flow of GPG, CHP, GB units, compressors and other gas loads at node i ; N_g is the total nodes number of NGS.

We consider the scenario that the presence of coupling unit, i.e., P2G inject a variety of gas components into the NGS [34]. This leads to unknown gas densities of the mixed gas in pipelines. Considering three common gas, such as hydrogen (H_2) and synthetic natural gas (SNG) produced by P2G, and traditional natural gas (NG) in NGS pipelines, the nodal balance equation of gas density after mixing the three gases is as follows:

$$\Delta \sigma_i = \sigma_i (f_i^{NG} + f_i^{H_2} + f_i^{SNG} + \sum_{j=1}^{N_g} \text{sign}_f(f_{ji}) \cdot f_{ji}) - [f_i^{NG} \sigma_{NG} + f_i^{H_2} \sigma_{H_2} + f_i^{SNG} \sigma_{SNG} + \sum_{j=1}^{N_g} \text{sign}_f(f_{ji}) \cdot (f_{ji} \sigma_j)] \quad \forall i = 1, 2, \dots, (N_g - 1), \quad (11)$$

where f_i^{NG} , $f_i^{H_2}$ and f_i^{SNG} are gas flow of NG, H_2 and SNG injected into node i , respectively; $\text{sign}_f(f_{ji})$ is a sign function of gas flow where $\text{sign}_f(f_{ji}) = 1$ if $f_{ji} \geq 0$, otherwise $\text{sign}_f(f_{ji}) = 0$; σ_{NG} , σ_{H_2} and σ_{SNG} are gas density of NG, H_2 and SNG, respectively; σ_i and σ_j are the specific gravity of mixed gas flowing out of node i and j , respectively. The gas density nodal balance equation is expanded based on the mass continuity of fluid [35]. In an ideal state, the product of the total flow and density of mixed gas flowing out of node i should be equal to the sum of the product of flow and density of each gas flowing into node i , expressed as $\sigma_{mix,i} \sum f_i = \sum (f_i \times \sigma_i)$.

2.3. District heating system (DHS)

The state of the district heat network is determined by thermal model and hydraulic model. The thermal model studies the heat power and temperature changes. The transferred heat power ϕ through a pipeline $i - j$ is given by

$$\phi_{ij} = C_p \dot{m}_{ij} (T_i^s - T_j^r), \quad (12)$$

$$T_i = (T_j - T_a) \psi_{ij} + T_a, \quad \psi_{ij} = e^{-\left(\frac{U_h L_h}{C_p \dot{m}_{ij}}\right)}, \quad (13)$$

where C_p is the water specific heat capacity; $\dot{m}_{ij}$ is the mass flow through pipeline $i - j$; T^s , T^r and T_a are the supply temperature, return temperature and environment temperature, respectively; ψ_{ij} is the temperature drop coefficient of pipeline ij ; U_h and L_h are the heat transition coefficient and length of the pipeline, respectively. For the simulation that multiple water flows with different temperatures converge at a node, the mixing temperature at the node is calculated as follows [36]

$$\left(\sum m_{out,i} \right) T_{out,i} = \sum (m_{in,i}) T_{in,i}, \quad (14)$$

where $m_{out,i}$ and $T_{out,i}$ ($m_{in,i}$ and $T_{in,i}$) are the mass flow and temperature of water flowing out (injecting into) the node i , respectively.

The hydraulic model studies the head losses h_f along the pipeline, which is calculated as

$$h_f = K_h \dot{m} |\dot{m}|, \quad (15)$$

where K_h is the resistance coefficient of the pipeline.

The steady-state equilibrium point for DHS should satisfies the Kirchhoff's law. It implies that the heat power injected into node i is equal to the heat power consumed at this node, and the sum of the pressure head loss of water flowing in a closed loop is zero. In this way, the balance equations for the DHS are given by:

$$\Delta \phi_i = \phi_i^{CHP} + \phi_i^{EB} + \phi_i^{GB} - \phi_i^{HL} - \sum_{j=1}^{N_h} \phi_{ij} \quad \forall i = 1, 2, \dots, (N_h - 1), \quad (16)$$

$$\Delta h_{f,i} = \sum_{j=1}^{n_{pipe}} B_{ij} h_{f,j} \quad \forall i = 1, 2, \dots, (N_{loop} - 1), \quad (17)$$

$$\Delta T_i^s = \sum (m_{in,i}) T_{in,i}^s - (\sum m_{out,i}) T_{out,i}^s \quad \forall i = 1, 2, \dots, (N_h - 1), \quad (18)$$

$$\Delta T_i^r = \sum (m_{in,i}) T_{in,i}^r - (\sum m_{out,i}) T_{out,i}^r \quad \forall i = 1, 2, \dots, (N_h - 1), \quad (19)$$

where ϕ_i^{CHP} , ϕ_i^{EB} and ϕ_i^{GB} are the heat power of CHP, EB and gas boiler (GB) units injected into node i , respectively; ϕ_i^{HL} is the consumed heat power of heat load; N_h is the total nodes number of DHS; n_{pipe} is the total pipelines number on loop i in DHS.

2.4. Interdependencies among EPS, NGS and DHS

The gas-fired CHP and GPG units consume gas to produce electricity power [37]. The relationships between the volume flow required in NGS and the active power output in EPS are expressed as

$$f_i^{CHP} = \frac{3.412}{Ghvi} \left(\frac{P_i^{CHP} + \phi_i^{CHP}}{\eta_{CHP}} \right), \quad (20)$$

$$f_i^{GPG} = \frac{1}{Ghvi} (\alpha_i^G (P_i^{GPG})^2 + \beta_i^G P_i^{GPG} + \gamma_i^G), \quad (21)$$

where α_i^G , β_i^G and γ_i^G are heat rate coefficients of GPG units at node i ; η_{CHP} is the conversion; Ghv is the gross heating value of gas at node i .

P2G technology converts power into gas via two processes [38]. In the first process, water is electrolyzed into H_2 that can be injected into gas network in a limit range. In second process, H_2 combines CO_2 to form CH_4 , known as synthetic natural gas (SNG), which can be directly injected into the gas network without restriction [38]. The products in two processes are different, but can be understood as similar processes with different efficiencies. Thus, a universal formula used to express the relationship between P_i^{P2G} and f_i^{P2G} can be given by

$$f_i^{P2G} = \left(\frac{3600\eta_{P2G}}{Ghvi} \right) P_i^{P2G}, \quad (22)$$

where η_{P2G} is the energy conversion efficiency.

For a moto-compressor, the consumed active power can be calculated by [13]

$$P_i^{com} = \frac{745.7 \times 10^{-6}}{3600} H P_i^{com}. \quad (23)$$

The GB and EB consume electricity and gas to produce heat power, respectively [39]. The models are as follows

$$f_i^{GB} = 3.412 \frac{\phi_i^{GB}}{\eta_{GB} Ghvi}, \quad (24)$$

$$P_i^{EB} = \frac{\phi_i^{EB}}{\eta_{EB}}, \quad (25)$$

where η_{GB} and η_{EB} are the energy conversion efficiency of GB and EB unit.

It is worth noting that the energy flow of coupled units is interactive, which is related to the Ghv. In traditional models, the gas composition is seen as a single component, and the Ghv is set to a fixed value. However, the injection and mixture of different composition gas sources directly change the gas characteristic-Ghv. The different gas with different Ghv will result in significant differences in the energy delivered to the loads. Therefore, the steady-state equilibrium point of Ghv is specified in this paper, so as to avoid the erroneous assessment of stored energy in gas pipeline. The balance equation used for the updating nodal Ghv is:

$$\begin{aligned} \Delta Ghv_i = & Ghv_i (f_i^{NG} + f_i^{H_2} + f_i^{SNG} + \sum_{j=1}^{N_G} \text{sign}_f(f_{ji}) \cdot f_{ji}) \\ & - [f_i^{NG} Ghv_{NG} + f_i^{H_2} Ghv_{H_2} + f_i^{SNG} Ghv_{SNG} \\ & + \sum_{j=1}^{N_G} \text{sign}_f(f_{ji}) \cdot (f_{ji} Ghv_j)] \quad \forall i = 1, 2, \dots, (N_g - 1), \end{aligned} \quad (26)$$

Table 1

Different node type and unknown variables.

Subsystem	Node type	Known	Unknown	Formulation
EPS	Reference	V, θ	$\tilde{P}, Q$	(5)–(6)
	PV	P, V	θ, Q	
	PQ	P, Q	θ, V	
	$\tilde{P}V$	V	$\tilde{P}, \theta, Q$	
	$\tilde{P}Q$	Q	$\tilde{P}, \theta, V$	
NGS	Slack	π, σ, Ghv	f	(10)–(11), (26)
	fL	f	π, σ, Ghv	
DHS	Slack	T^s	m, ϕ, T^r	(16)–(19)
	ϕT^s	ϕ, T^s	m, T^r	
	ϕT^r	ϕ, T^r	m, T^s	

where $Ghvi$ and $Ghvj$ are gross heating value of mixed gas flow out of node i and j , respectively. $Ghvi_{NG}$, $Ghvi_{H_2}$ and $Ghvi_{SNG}$ are the gross heating value of the NG, H_2 and SNG, respectively. The nodal balance equation of Ghv is derived based on the volume fractions and Ghv of each individual gases [15]. For brevity, the Ghv of a mixture gas leaving a node is the average of all incoming pipelines' gas characteristic considering flows as their weighting factors. It can be expressed as $X_{out} = \sum (f_{in} X_{in}) / \sum f_{in}$.

3. Robust energy flow calculation method

3.1. Problem formulation

The steady-state EFC is equivalent to solve equilibrium point of IES, which can be obtained by combing the balance equations of EPS, NGS, DHS and the interdependencies models in Section 2. Thus, the integrated formulation describing the energy flow problem is summarized as

$$F(X) = [\Delta P, \Delta Q, \Delta f, \Delta \sigma, \Delta Ghv, \Delta(\phi, h), \Delta T^s, \Delta T^r]^T, \quad (27)$$

where $F(X)$ is the vector of all equations to be solved for EFC. The unknown variables are defined as:

$$X = [\Delta P_b, \theta, V, \pi, \sigma, Ghv, m, T^s, T^r]^T, \quad (28)$$

The number of unknowns variables to be solved is equal to the number of equations, which is depending on node types. Table 1 summarizes the node type of different networks and unknown variables. The total number of unknown variables is $(N_e + N_{pq} + 3N_g + N_{pipe} + 2N_h - 5)$.

3.2. Consecutive damping factor empowered robust energy flow calculation method

According to (27), the solution of energy flow problem is equivalent to the solution of nonlinear equations:

$$F(X) = 0. \quad (29)$$

where $F(X)$ consists of (5)–(6), (10)–(11), (16)–(19), (26). The solution X starts from an initial guess X_0 . In convergence case, for each iteration k , there is a search step Δ_k such that $F(X_k + \Delta_k)$ is less than $F(X_k)$. Therefore, to describe the behavior of F in the neighborhood of the point X_k , a function $\Psi(\Delta_k)$ is given by

$$F(X_k + \Delta_k) \simeq \Psi(\Delta_k). \quad (30)$$

By expanding the left-hand of (30) with the second order Taylor expansion, the problem of nonlinear equations is translated into the following subproblem:

$$\min |F(X_k) + \Delta_k^T g_k + \frac{1}{2} \Delta_k^T h_k \Delta_k|, \quad (31)$$

where $\mathbf{g}_k$ and $\mathbf{h}_k$ are the gradient and Hessian of F at $\mathbf{X}_k$, respectively. The solution of this minimization problem needs to satisfy

$$(\mathbf{h}_k + \lambda \mathbf{I}) \Delta_k = -\mathbf{g}_k. \quad (32)$$

Then, the search step Δ_k at each iteration is conducted by

$$\begin{aligned} \Delta_k &= -(\mathbf{h}_k + \lambda \mathbf{I})^{-1} \mathbf{g}_k \\ &= -(J(\mathbf{X}_k)^T J(\mathbf{X}_k) + \lambda \mathbf{I})^{-1} J(\mathbf{X}_k)^T F(\mathbf{X}_k), \end{aligned} \quad (33)$$

where $\mathbf{I}$ is identity matrix. λ refers to damping factor. $\mathbf{J}$ is the Jacobian matrix, and its expression for this particular problem is given in (34), shown as

$$\mathbf{J} = \begin{bmatrix} \frac{\partial \Delta P}{\partial P_d} & \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} & \frac{\partial \Delta P}{\partial \pi} & \frac{\partial \Delta P}{\partial \sigma} & \frac{\partial \Delta P}{\partial Ghv} & \frac{\partial \Delta P}{\partial \dot{m}} & 0 & \frac{\partial \Delta P}{\partial T^r} \\ 0 & \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial \theta} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial \Delta f}{\partial P_d} & \frac{\partial \Delta f}{\partial \theta} & \frac{\partial \Delta f}{\partial V} & \frac{\partial \Delta f}{\partial \pi} & \frac{\partial \Delta f}{\partial \sigma} & \frac{\partial \Delta f}{\partial Ghv} & \frac{\partial \Delta f}{\partial \dot{m}} & 0 & \frac{\partial \Delta f}{\partial T^r} \\ 0 & 0 & 0 & 0 & \frac{\partial \Delta \sigma}{\partial \sigma} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial \Delta Ghv}{\partial Ghv} & 0 & 0 & 0 \\ 0 & \frac{\partial [\Delta(\phi, h)]}{\partial \theta} & \frac{\partial [\Delta(\phi, h)]}{\partial V} & 0 & 0 & 0 & \frac{\partial [\Delta(\phi, h)]}{\partial \dot{m}} & \frac{\partial [\Delta(\phi, h)]}{\partial T^s} & \frac{\partial [\Delta(\phi, h)]}{\partial T^r} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial \Delta T^s}{\partial T^s} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial \Delta T^r}{\partial T^r} \end{bmatrix} \quad (34)$$

where the off-diagonal block matrices represent the interdependencies between sub-network. For instance, $\partial \Delta f / \partial \Delta P_d$ is derivative of the gas flow mismatches with respect to the active power slack. It exists because the gas flow consumption varies with the active power output of gas-fired generator. The matrices $[\partial \Delta P / \partial \pi, \partial \Delta P / \partial \sigma, \partial \Delta P / \partial Ghv]$ in the first row represent the impact of natural gas on electric network. They are nonzero if moto-compressors are loads of the electric networks because the output of the electric compressor varies with the state variables of the gas network, which leads to the active power fluctuation of the electric network. The matrices $[\partial \Delta P / \partial \dot{m}, 0, \partial \Delta P / \partial T^r]$ represent the impact of heat network on electric network. They are exist if the slack bus of heat network is supplied by electric-boiler. It is worth noting that $\partial \Delta \sigma / \partial \sigma$ and $\partial \Delta Ghv / \partial Ghv$ are new diagonal items when considering multi-gas composition. Meanwhile, $\partial \Delta f / \partial \sigma$ and $\partial \Delta f / \partial Ghv$ are not zero due to the effect of SG and GCV on volume flow.

By controlling λ , we can adjust the direction and size of the search step. Firstly, for $\lambda > 0$, the coefficient matrix $\mathbf{h}_k + \lambda \mathbf{I}$ is positive definite, which can be used to prevent Δ_k from being too large when $\mathbf{J}$ is nearly singular. Hence, using the damping factor λ can make the solution of the search direction more stable. Secondly, the different value of λ determines the size of the search step Δ_k , which is discussed in following two categories:

(a) For a large λ within $\lambda > 0$, Δ_k can be approximated by

$$\Delta_k \approx -\frac{1}{\lambda} J(\mathbf{X}_k)^T F(\mathbf{X}_k). \quad (35)$$

Δ_k is a small step in the descent direction. This is suitable for the case if the current iterate is far from the truth solution.

(b) For a small λ within $\lambda > 0$, Δ_k can be approximated by

$$(J(\mathbf{X}_k)^T J(\mathbf{X}_k) + \lambda \mathbf{I}) \Delta_k = -J(\mathbf{X}_k)^T F(\mathbf{X}_k). \quad (36)$$

This is also a descent direction for F because

$$-\Delta_k^T (J(\mathbf{X}_k)^T J(\mathbf{X}_k) + \lambda \mathbf{I}) \Delta_k = \Delta_k^T J(\mathbf{X}_k)^T F(\mathbf{X}_k) < 0. \quad (37)$$

The form in (37) is quadratic convergence, which can ensure fast convergence. Hence, the small value of λ is suitable for the case of final stage, during which the current iteration is close to the true value.

In summary, the choice of damping factor λ directly influences the direction and size of search step. To ensure the solvability of solutions, λ should be adaptively updated at each iteration according to the quality of the computed step Δ_k . To reasonably assess the quality of

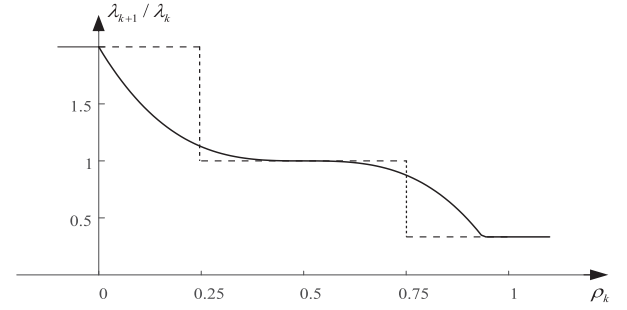


Fig. 1. The updating approach of damping factor.

the computed step, a gain ratio ρ considering the decrease of actual function F and predicted function Ψ , is defined as:

$$\rho_k = \frac{F(\mathbf{X}_k) - F(\mathbf{X}_k + \Delta_k)}{\Psi(0) - \Psi(\Delta_k)}. \quad (38)$$

The value of ρ_k reflects the approximation degree of $\Psi(\Delta_k)$ to $F(\mathbf{X}_k + \Delta_k)$. If ρ_k is small or negative, this indicates that the current operational point of $\Psi(\Delta_k)$ is a poor approximation of $F(\mathbf{X}_k + \Delta_k)$ for the computed step Δ_k . In this situation, the step Δ_k will drive $\mathbf{X}_k$ towards an untrusted region. Then, we should increase the damping factor λ to increase the penalty and to avoid a failed step in next iteration. Conversely, if ρ is large. It indicates that the $\Psi(\Delta_k)$ is a desirable approximation of $F(\mathbf{X}_k + \Delta_k)$ for step Δ_k . In this situation, we should reduce the damping factor λ so as to increase the step length in next iteration. Thus, the convergence rate can be increased. Based on the discussion above, a specific updating strategy of damping factor λ_k extended from [23] is to follow the performance of Δ_k at k th iteration. The strategy is given by:

if $\rho_k \geq 0$

$$\lambda_{k+1} = \lambda_k \times \max\left\{\frac{1}{u}, 1 - (\varpi - 1)(2\rho - 1)^\rho\right\}; \quad \tau_k = \varpi \quad (39)$$

else if $\rho_k < 0$

$$\lambda_{k+1} = \lambda_k \times \tau_k; \quad \tau_{k+1} = u \times \tau_k$$

where $u > 1$, $\varpi > 1$ and ρ is odd integer. The proposed updating approach enables consecutively updating as the change of ρ_k . For instance, when $u = 3$, $\varpi = 2$ and $\rho = 3$, the change curve of damping factor obtained by performing (39) is shown in Fig. 1. It can be seen that the solid curve is bounded by 0.5. When $\rho_k < 0.5$, it is considered as a bad approximation, in which $\lambda_{k+1}/\lambda_k > 1$. It means that the current iteration is untrustworthy. Hence, the penalty is increased with a large damping factor. Moreover, the smaller ρ_k is, the larger the damping factor λ_{k+1} is. When $\rho_k > 0.5$, it is considered as a desirable approximation, in which $\lambda_{k+1}/\lambda_k < 1$. It means that the current iteration is trustable. Then, the penalty is decreased with smaller damping factor. The traditional updating strategy for damping factor is nonconsecutive (see from the dashed line in Fig. 1), which only takes three fix values of 2, 1, 1/3 for λ_{k+1}/λ_k associated with the intervals $0 \sim 0.25$, $0.25 \sim 0.75$ and $0.75 \sim 1$, respectively. The nonconsecutive changes of three intervals may lead to flutter, and further slow down convergence.

By performing the adaptive damping factor λ , a reasonable search step Δ_k can be determined. Next, at $(k+1)$ th iteration, $\mathbf{X}$ is updated as

$$\mathbf{X}_{k+1} = \begin{cases} \mathbf{X}_k + \Delta_k & \rho_k > \rho_0 \\ \mathbf{X}_k & \rho_k < \rho_0 \end{cases} \quad (40)$$

where $0 < \rho_0 < 1$, and is used for determining whether to accept the updating of the current step Δ_k .

The above procedure is repeated until the required convergence accuracy ε is achieved. The stopping criteria is defined as

$$\|J(X_k)^T F(X_k)\| \leq \varepsilon. \quad (41)$$

Finally, the proposed consecutive damping factor empowered EFC algorithm is summarized as the following steps:

Step (1): Given $X_0 \in R^n$, $\varepsilon > 0$, $k = 0$, $u > 1$, $\varpi > 1$ and an odd integer ρ , $0 < \rho_0 < 1$.

Step (2): If (41) is satisfied, then stop, and the solution of energy flow is obtained. Otherwise, go to Step (3).

Step (3): Calculate search step at k_{th} iteration using damping method (33).

Step (4): Calculate the gain ratio ρ_k by using (38), and adjust the damping factor by using approach (39).

Step (5): Decide whether to accept the search step Δ_k by using (40).

Step (6): $k = k + 1$; return to Step (2).

3.3. Convergence analysis

The convergence conditions and properties of the proposed EFC algorithm are discussed in this section. We make the following assumptions:

Assumption 1. $F(X)$ is continuously differentiable, and both the $F(X)$ and $J(X)$ are Lipschitz continuous, i.e. there exists constant $C > 0$, such that

$$\|F(Y) - F(X)\| \leq C\|Y - X\|, \forall X, Y \in R^n, \quad (42)$$

$$\|J(Y) - J(X)\| \leq C\|Y - X\|, \forall X, Y \in R^n. \quad (43)$$

Lemma 1 ([40]). Under the conditions of Assumption 1, there exists a positive constant $L > 0$, such that

$$\|F(Y) - F(X) - J(X)(Y - X)\| \leq C\|Y - X\|^2, \quad (44)$$

$$\|F(X_{k+1})\|^2 \leq \|F(X_k)\|^2 - L \frac{(F(X_k)^T J(X_k) \Delta_k)^2}{\|\Delta_k\|^2}. \quad (45)$$

On the one hand, the energy flow equations can be continuously differentiable [11]. On the other hand, the state variables in EFC are bounded, so it can be prove that $F(X_k)$ and $J(X_k)$ are Lipschitz continuous [10]. Hence, the Lemma can be obtained.

Theorem 1. Suppose X_k is a sequence generated by proposed algorithm and X^* is the solution of the Eqs. (29). If Assumption 1 holds, the sequence generated by proposed algorithm satisfies

$$\lim_{k \rightarrow \infty} \|J(X_k)^T F(X_k)\| = 0. \quad (46)$$

Proof. Δ_k is a search step generated by the proposed algorithm. The Δ_k satisfies

$$\|F(X_k + \Delta_k)\| \leq \eta \|F(X_k)\|, \quad (47)$$

where $\eta \in (0, 1)$. It follows from (45) and (47) that

$$\sum_{k=0}^{\infty} \frac{(F(X_k)^T J(X_k) \Delta_k)^2}{\|\Delta_k\|^2} \leq +\infty. \quad (48)$$

When $F(X)$ converges to 0, $F(X_k) = F(X^*) = 0$; otherwise, $\|F(X)\| \rightarrow \zeta > 0$. According to the above inequality, the definition of Δ_k in (33) and $\|F(X_k)\| \geq \zeta \geq 0$ imply that

$$\begin{aligned} & (F(X_k)^T J(X_k) \Delta_k)^2 \\ &= (\Delta_k^T (J(X_k)^T J(X_k) + \lambda I) \Delta_k)^2 \\ &\geq \zeta^2 \|\Delta_k\|^4. \end{aligned} \quad (49)$$

Next, by using (48) and (49), we have

$$\lim_{k \rightarrow \infty} \|\Delta_k\| = 0. \quad (50)$$

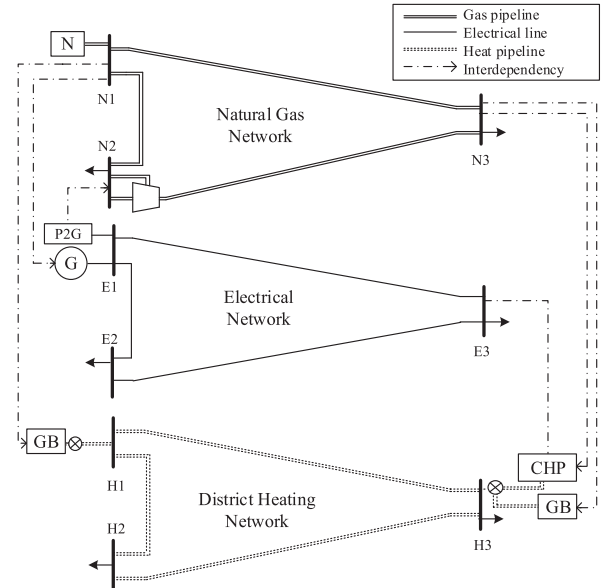


Fig. 2. Schematic diagram of the small IES.

Eqs. (35), (36) and (50) imply that $\|J(X^*)^T F(X^*)\| = 0$ at any accumulation point X^* of X_k . Hence, the sequence X_k can converge to the final point. This implies that the proposed algorithm achieves a global convergence.

4. Case studies and simulations

4.1. The effectiveness of consecutive updating strategy of damping factor

To test the performance of the proposed consecutive updating strategy of damping factor, the formulation of a 3-3-3 IES, composed of 3-bus power system, 3-node natural gas network and 3-node heating network, is solved. The parameters of proposed strategy are set to $u = 3$, $\varpi = 2$, $\rho = 3$, $\rho_0 = 0.01$ and $\varepsilon = 1e-6$. The schematic of this test system is shown in Fig. 2. The system parameters are the same as those in [13]. The flat start is applied for EPS and DHS. The voltages of EPS are initialized at $V = 1$ p.u. and $\theta = 0^\circ$ for all nodes. The DHS is initialized at $m = 100$ kg/s for all pipelines, and $T^s = 120^\circ\text{C}$, $T^r = 50^\circ$ for all nodes. For NGS, the difference start is applied for pressure, which are initialized at $N1 = 50$ bar, $N2 = 40$ bar and $N3 = 49$ bar. σ and G_{hv} are initialized at the values of main gas source, where $\sigma = 0.6048$ and $G_{hv} = 41.04$ MJ/m³.

Fig. 3 shows the performance of iteration process with proposed damping factor updating strategy. It can be seen that the damping factor λ changes smooth and gradual with the increase of iteration numbers. At the final convergence stage, the $\|J(X_k)^T F(X_k)\|$ fast converge with a small value of λ . As a comparison, the results of the non-consecutive adaptive factor updating method in [10] are shown in Fig. 4. As we can see, from the 14th to 23rd iterations, every decrease in the λ is followed by another increase. This is because the non-consecutive updating strategy causes a sudden change of the damping factor and leads to a overshoot or undershoot adjustment, which requires additional iterations to correct. Additionally, the convergence conditions $\|J(X_k)^T F(X_k)\|$ are accompanied with oscillations, resulting in slow down convergence. The iteration numbers is 27 for non-consecutive damping factor updating strategy while that of the proposed method is 22.

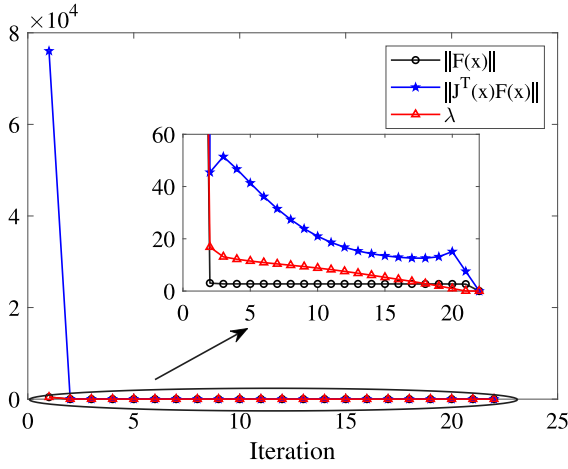


Fig. 3. Performance of iteration process with proposed damping factor updating strategy.

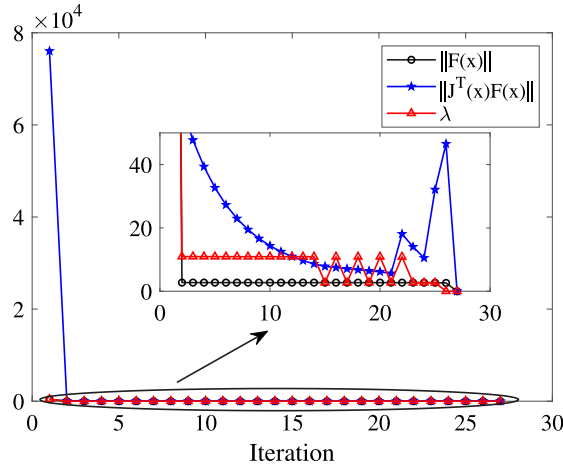


Fig. 4. Performance of iteration process without proposed damping factor updating strategy.

4.2. Robustness analysis

In this section, we demonstrate the performance of the proposed EFC method on two typical IES test systems. The integrated Newton-Raphson (INR) [13,36] is taken as a benchmark for comparison, since it is suitable for integrated electricity-gas-heat systems. Firstly, four types of operational modes of 3-3-3 IES are analyzed: Case (1) well-conditioned operation mode; Case (2) inappropriate initial values; Case (3) heavy loads; Case (4) ill-conditioned operational mode. By performing the proposed method, the EFC results are listed in Table 2–Table 6.

Case (1) well-conditioned operation mode: Taking the values in Section 4.1 as initial conditions, the iteration number calculated by proposed method and traditional INR method are 22 and 18, respectively. The EFC results are the same between the two methods. The detailed EFC results are summarized in Tables 2 and 3 with participation factors of 0.5 for both two generators. The reactive power limits for the two generators are ± 200 and ± 50 MVar, respectively. The correctness of the results can be verified by balance equations (27) and Kirchhoff's law. For instance, in gas node 3, the injected gas flow is the sum of the flow transmitted on pipeline 1–3 (f_{13}) and (f_{23}). The value is 25.7147, which is exactly equal to the consumed flow (the sum of f_3^{CHP} , f_3^{GB} and f_3^{GL}). In addition, the values of σ and $G_{h,v}$ at node 3 are verified by the $\frac{X_1 f_{13} + X_2 f_{23}}{f_{13} + f_{23}}$, which is consistent with (11) and (26).

Case (2) inappropriate initial values: Traditional EFC methods are very sensitive to the initial values. Inappropriate initial values may cause non-convergence. In this case, different initial values of voltages magnitude for EPS, nodal pressure for NGS, and mass flow for DHS are used to test the performance of proposed method. Table 4 reports the iteration number and mismatch at the final convergence under different initial values. It can be seen that the INR method has a lower iterations numbers for some well-condition situations, and enables to converge when the initial value of voltage is near to 1.0 p.u.. On the contrast, the proposed method has better robustness with a wide range of initial value, i.e., 0.5 p.u. $\sim$ 1.1 p.u. The equilibrium points of nodal voltage, nodal pressure and pipeline mass flow under different initial values are also shown in the Table 4. It can be seen that the proposed method can converge to the same equilibrium points under different initial values of gas network and heat network. Note that although the convergent values at 0.5 p.u. and 0.8 p.u. can be obtained by proposed method, it does not necessarily represent the realistic operation state.

It can be seen from Table 4 that the EFC is more sensitive to the initial pressures of NGS. This is because that the derivative $\partial \Delta f_i / \partial \pi_i$ in Jacobin matrix $\mathbf{J}$ (Eq. (34)) is related to the inverse of the difference $\pi_i^2 - \pi_j^2$ ($\partial \Delta f_i / \partial \pi_i = C_{ij} \pi_i / (\pi_i^2 - \pi_j^2)^{-0.5}$). When flat initialization used, the ill-conditioned elements such as singular Jacobian matrix caused by the $\pi_i^2 - \pi_j^2 = 0$ will be brought. In this scenario, the EFC solutions will fail to converge. To address this issue, the difference initialization widely replaces the flat initialization. However, as shown in Table 4, the difference initialization in INR method is only suitable for initial pressure (50.45,40.5), and still is not able to converge within 1000 iterations for (50,46,40) and (50,45,45). On the contrast, the proposed method can converge with fewer iterations under all initial pressure values.

Case (3) heavy loads: Heavy loads may make energy flow calculation algorithms unsolvable. Table 5 compares the iteration numbers and mismatches between our proposed method and the INR method under different expansion factors of loads. It can be seen that the INR method converges with lower iteration numbers when the expansion factors reach 1.5 and 2.5, and diverge when the expansion factor reaches 3.5. On the contrast, the proposed method can converge to small values under all expansion factors. Note that $\|F(X)\|_\infty = 8.6274e-3$ within 200 iterations when factor is 3.5. Although the mismatch does not meet the accuracy requirement $1e-6$, it can still be considered as a valuable approximation in practice, compared with the diverge solution calculated by traditional INR method.

The equilibrium points under different load levels are also shown in the Table 5, where V , π and m represent the nodal voltage, nodal pressure and pipeline mass flow vector, respectively. As the load level increases, the voltage and pressure decrease to allow for more power and gas flow to the electric and gas loads. Additionally, the mass flow increases to transfer more thermal power to the heat load. It is worth noting that, due to the reactive power limitation, the voltage magnitude of node 3 changes from its fixed value of 1 to a free value when the load expands to 3.5 times the original. In addition, the pressure may include imaginary numbers. This is because the pressure in the gas network is no longer sufficient to support normal pressure level. However, it is important to note that this operating state is not permitted in real-world scenarios. Therefore, the proposed method is helpful in obtaining more operational information.

Case (4) ill-conditioned operation modes: In this scenario, the three break line faults (the power line or pipe line is out of service) are respectively used to simulate the ill-conditioned operational modes. The comparison results between the proposed method and the INR method are reported in Table 6. For the line 1–3 fault in EPS and NGS, the INR method achieves satisfactory accurate solutions after 17 and 9 iterations, respectively, while the proposed method converges to smaller values after 27 and 13 iterations. For the pipeline 1–3 fault in DHS, the INR method fails to converge even after 1000 iterations,

Table 2

The EFC results of the 3-3-3 IES under well-conditioned operation mode in Case 1.

Node	Voltage		Pressure	Gas characteristic		Temperature		Line	Power flow	Gas flow	Mass flow
i	V_i (p.u.)	θ_i (°)	π_i (bar)	σ	Ghv (MJ/m ³)	T_i^s (°C)	T_i^r (°C)	$i-j$	$P_{ij} + jQ_{ij}$ (MW, MVar)	f_{ij} (Mm ³ /h)	$\dot{m}_{ij}$ (kg/s)
1	1.0600	0.0000	50.0000	0.6048	41.0400	120.0000	48.6805	1-2	20.3623+j15.4868	28.8909	64.6943
2	0.9814	-5.1883	41.4462	0.6026	40.7184	119.0370	50.0000	1-3	9.9187+j11.9079	4.8155	31.4533
3	1.0000	-2.3595	49.7862	0.6030	40.7786	123.5410	49.5339	2-3	-9.9289-j2.4255	20.8992	-56.5335

Table 3

The interdependencies and load of the 3-3-3 IES under well-conditioned operation mode in Case 1.

Node	EPS (MW)				NGS (Mm ³ /h)						DHS (MW)		
i	P_i^{GPG}	P_i^{CHP}	P_i^{P2G}	P_i^{EL}	f_i^{GPG}	f_i^{CHP}	f_i^{P2G}	f_i^{GB}	f_i^{Com}	f_i^{GL}	ϕ_i^{CHP}	ϕ_i^{GB}	ϕ_i^{HL}
1	30.2810	0	0.0416	0	6.8298	0	0	2.9802	0	0	0	28.6768	0
2	0	0	0	30	0	0	2.8	0	0.7128	10.0790	0	0	35
3	0	30.2810	0	30	0	5.1681	0	0.4184	0	20.1282	25.0120	4	20

Table 4

The iterations and mismatches of The 3-3-3 IES under inappropriate initial values in Case 2.

Network	Different	Proposed method			INR method		
(initial variable, units)	initial values	Iteration number	Mismatch ($\ F(X)\ _\infty$)	Equilibrium point	Iteration number	Mismatch ($\ F(X)\ _\infty$)	Equilibrium point
EPS	1.1	23	6.0350e-7	(1.1660,1.0897,1.1)	18	4.6111e-7	(1.1660,1.0893,1.1000)
(Voltage for all buses, p.u.)	0.8	22	6.0350e-7	(0.8480,0.7588,0.8)	18	4.6111e-7	(0.8480,0.7587,0.8000)
	0.5	26	9.3416e-7	(0.5000,0.2662,0.5)	> 1000	Diverge	-
NGS	50,45,40.5	33	3.9705e-7	(50,40.8292,49.6006)	45	3.8979e-7	(50,41.0315,49.7476)
(Pressure at each node, bar)	50,46,40	33	5.0390e-7	(50,40.8292,49.6006)	> 1000	Diverge	-
	50,45,45	37	4.4542e-7	(50,40.8292,49.6006)	> 1000	Diverge	-
DHS	50,50,-50	24	5.3164e-7	(64.6943, 31.4533,-56.5335)	18	4.6111e-7	(64.6943, 31.4533,-56.5335)
(Mass flow at each pipe, kg/s)	60,50,-30	30	8.6782e-7	(64.6943, 31.4533,-56.5335)	18	4.6111e-7	(64.6943, 31.4533,-56.5335)
	60,30,50	47	3.7410e-7	(64.6943, 31.4533,-56.5335)	25	4.6111e-7	(64.6943, 31.4533,-56.5335)

Table 5

The iterations and mismatches of the 3-3-3 IES under heavy loads in Case 3.

Type	Expansion	Proposed method			INR method		
	factor of loads	Iteration number	Mismatch ($\ F(X)\ _\infty$)	Equilibrium point	Iteration number	Mismatch ($\ F(X)\ _\infty$)	Equilibrium point
Heavy load	1.5	19	4.7854e-7	V(1.06,0.9502,1) π (50, 38.7143,47.2097) m(108.7971,81.1442,72.4736)	15	6.7751e-7	V(1.0600,0.9499,1) π (50, 38.7143,47.2097) m(108.7971,81.1442,72.4736)
	2.5	28	2.28122e-7	V(1.06,0.8703,1) π (50,24.1712,37.2125) m(201.7890,175.6482,99.3303)	16	5.66794e-7	V(1.06,0.8766,1) π (50,24.1712,37.2125) m(201.7890,175.6482,99.3303)
	3.5	200	8.4845e-2	V(1.06,0.6239,0.8419) π (50,13.527*i,24.939*i) m(296.3354,268.9108,124.5057)	>1000	Diverge	V- π - m-

Table 6

The iterations and mismatches Of the 3-3-3 IES under Ill-Conditioned operation modes in Case 4.

Type	Fault	Proposed method			INR method		
	in different networks	Iteration number	Mismatch ($\ F(X)\ _\infty$)	Equilibrium point	Iteration number	Mismatch ($\ F(X)\ _\infty$)	Equilibrium point
Ill-conditioned operation mode	line 1-3 fault in EPS	27	6.0350e-7	V(1.06,0.9789,1) π (50,40.8292,49.6006) m(64.6943, 31.4533,-56.5335)	17	4.6111e-7	V(1.0600,0.9783,1) π (50,40.8292,49.6006) m(64.6943, 31.4533,-56.5335)
	Pipeline 1-3 fault in NGS	13	8.4826e-8	V(1.06,0.9814,1) π (50,36.1565,39.6002) m(64.6943, 31.4533,-56.5335)	9	1.1790e-7	V(1.06,0.9808,1) π (50,37.0363,41.5312) m(64.6943, 31.4533,-56.5335)
	Pipeline 1-3 fault in DHS	200	3.2073e-2	V(1.06,0.9814,1) π (50,41.0315,49.7476) m(74.5481, -, 37.9554)	>1000	Diverge	V- π - m-

while the proposed method converges to $\|F(X)\|_\infty = 3.2073e-2$ within 200 iterations. Although $3.2073e-2$ does not meet the accuracy requirements, it can be a reference for the practical application compared

with the diverge solution obtained by using the traditional INR method. The equilibrium points under different line faults situations are also displayed in the Table 6. During system failure, the adjusted power

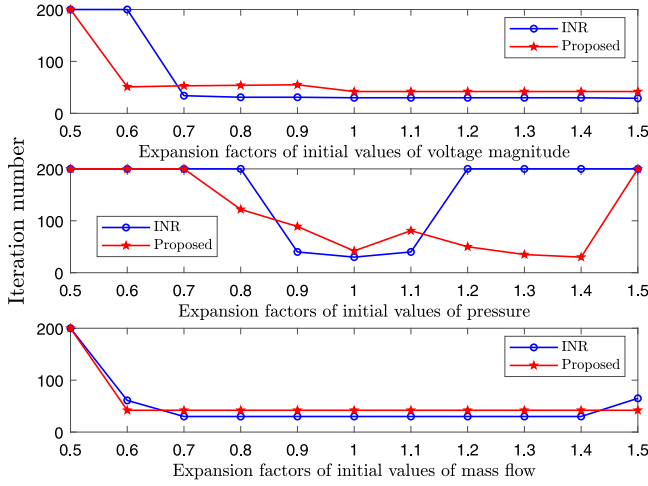


Fig. 5. Iterations of 118-20-14 IES under different initial values.

flow significantly deviates from the initial steady state. For instance, the mass flow through pipeline 2–3 changes from value ‘−56.5335’ to value ‘37.9554’, and the size and direction of mass flow both change accordingly.

In this case, the proposed method provides a desirable approximation for IES under various operation modes, not only including well-conditioned operation mode, but also including inappropriate initial values, heavy loads, and ill-conditioned operation modes. By capturing more value information from equilibrium points, this method helps to ensure the safe and efficient operation of the network.

To further verify the robustness and effectiveness of the proposed method. A large-scale IES, composed of 118-bus IEEE test system [20], 20-node Belgium natural gas network [41] and 14-node wide-area heating network [13], is used to test proposed EFC method. The inappropriate initial value and heavy loads mode are prone to bring ill-conditioned elements and cause unsolvable problem of EFC. Therefore, these modes are analyzed for large-scale IES. The EFC results are shown in Figs. 5–8.

Fig. 5 compares the iteration numbers under different initial values between two methods. The initial values including voltage magnitude, pressure and mass flow vary with the expansion factor. It can be seen that INR method converges with a narrower initial voltage value between 0.7 ~ 1.5, while the proposed method can still converge within 50 iterations when the initial voltage value is 0.6. Moreover, for the changes of initial pressure in NGS, the INR method can only converge when the expansion factor is close to 1, that is 0.9 ~ 1.1. However, the proposed method can converge well when the expansion factor is from 0.8 to 1.4, which has better robustness. For the changes of initial mass flow in DHS, the INR method and the proposed method have the same convergence performance. However, in the extreme factors (e.g., 0.6 and 1.5), the proposed method converges faster with a lower number of iterations than the INR method.

As shown in Fig. 5, the proposed method has better performance to handle the inappropriate initial pressure. To further observe the iteration process in ill-conditioned mode caused by inappropriate initial pressure, we display the mismatches of pressure under expansion factors 0.5, 0.6, 0.7 and 1.5 in Fig. 6. It can be seen that when the expansion factor equals 0.5, 0.6 and 1.5, the mismatches in INR method increase sharply with the increase of iterations. After the 16, 33 and 22 iterations, the mismatches becomes null, which fails to converge. When the expansion factor is 0.7, the mismatch in INR method also increases with the iteration and reaches 2.7×10^9 after 200 iterations. On the contrast, the mismatches of pressure in proposed method under

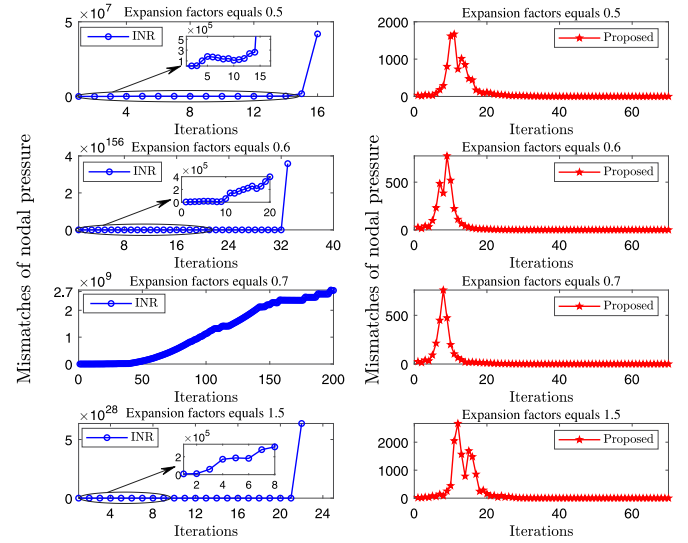


Fig. 6. Iteration process of Mismatches of nodal pressure.

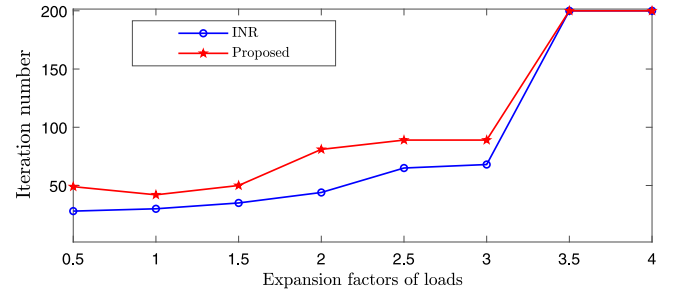


Fig. 7. Iterations of 118-20-14 IES under different loads.

expansion factors 0.5, 0.6, 0.7 and 1.5 all converge to a small value near to 0.

Fig. 7 compares the iterations under different level loads of the two methods. The loads in IES change with the expansion factor. It can be seen that both the proposed method and the INR method converge within 200 iterations under the expansion factor in the range 0.5~3. To further observe the iteration process under heavy loads mode, the mismatches of original function $F(x)$ under large expansion factors 3.5 and 4 are shown in Fig. 8. It can be seen that the heavy load causes cyclical oscillations after several iterations by using INR method, and finally fails to converge until 200 iterations. In contrast, the proposed method converges to a positive value less than 1 within 10 iterations when the expansion factors is 3.5. Although the value cannot meet the accuracy requirement $1e-6$, it can still be considered as a valuable approximation, compared with the oscillations results in INR method.

In conclusion, for the large IES, the process in traditional method fails to converge under some extreme scenarios, whereas the proposed method can converge or to give valuable approximation. The results further demonstrate the effectiveness of the proposed method for solving non-convergence energy flow. Overall, the proposed method possesses better robustness and universality in different operational modes, making it a valuable tool for IES analysis and optimization.

4.3. Computation time

The average computational time of different cases in Sections 4.1 and 4.2 is presented in Table 7. It can be seen that the computational time varies with different operation modes. This is because when the system works under different operation modes, the scalability and

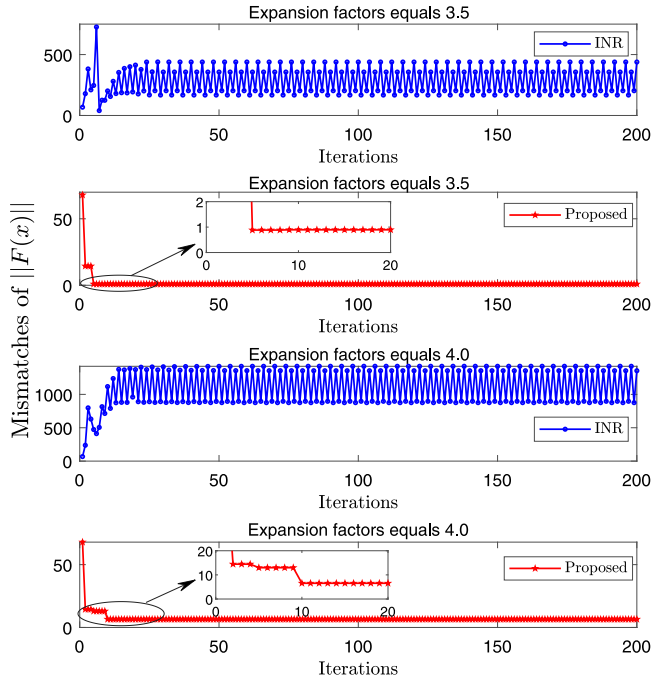


Fig. 8. Iteration process of Mismatches of $\|F(x)\|$.

Table 7

The computation time for energy flow under different cases.

IES	Mode	Proposed method	INR
3-3-3	Well-conditioned	0.6157 s	0.4843 s
	Inappropriate Initial Values	0.9430 s	1.0941 s
	heavy loads	0.7344 s	0.9438 s
	Fault in EPS	0.9375 s	0.4688 s
	Fault in NGS	0.5156 s	0.4063 s
	Fault in DHS	7.8125 s	9.3750 s
118-20-14	Well-conditioned	2.0193 s	1.7628 s
	Inappropriate Initial Values	4.0156 s	12.3281 s
	heavy loads	4.0625 s	4.9531 s

topology of the system can change, requiring a recalculation of energy flow.

Compared with the INR method, the proposed method consumes more time under well-conditioned modes, while consume less time under some ill-conditioned or unsolvable modes. This is because the INR method is quadratic convergence under well-conditioned modes, i.e., variables X are close to the true value X^* , while the proposed method considers jointly direction and size of search step for a secure search under all situations. Meanwhile, the consecutive adaptive damping factor updating approach is beneficial to avoid flutter of iteration. Hence, the proposed method fits well with non-convergence EFC problem in IES.

4.4. Complexity analysis

A complexity analysis based on the Big-O expression of the proposed method is presented in this section. There are several main elements in EFC that affect the time complexity: (i) calculation of the Jacobian matrix $J(X_k)$ in (34). The dimension of the Jacobian matrix in IES is N_{IES} by N_{IES} , where N_{IES} is equal to $N_e + N_{pq} + 3N_g + N_{pipe} + 2N_h - 5$. The time complexity for calculating the $J(X_k)$ is $O(N_{IES}^2)$; (ii) multiplication of two N_{IES} dimensional matrices in (33). The time complexity to multiply 2 matrices with N_{IES} rows and N_{IES} columns is $O(N_{IES}^3)$; (iii) the inverse of the N_{IES} dimensional matrix. The time complexity to calculate the inverse matrix with N_{IES} rows and

N_{IES} columns is $O(N_{IES}^3)$; (iv) the number of iterations k converges to meet the error tolerance ϵ . Considering the above main calculation components, the time complexity of the proposed method is $O(kN_{IES}^3)$, which is consistent with the traditional INR method.

In conclusion, the consumption time of the methods are mainly decided by the iteration number k . It can be seen that in Section 4.2, the k in proposed method is always larger than that in INR method under well-conditioned mode, and normally smaller than that under inappropriate initial values mode, heavy load mode as well as ill-conditioned mode. Accordingly, the computation time in Section 4.3 varies with the iteration number.

5. Conclusion

In this paper, a novel robust energy flow calculation method is proposed for IES consisting of detailed electricity, natural gas, heat system and inter-dependencies. Numerical simulations are verified in different cases, the performance of the proposed method is as follows. First, the proposed method possesses better robustness and universality compared with the traditional INR method. It can converge under a wide range of initial value. This is especially true for NGS with sensitive initial pressures, as the selection range of initial pressures has tripled. In case where unsolvable scenarios occur, the proposed method can almost converge a positive value less than 1 to realize a valuable approximation under limited iteration number. Further, the cyclical oscillations and flutter in iteration process can be effectively avoided by using the proposed method, which generally has less computation time and faster convergence rate under some extremely operation modes. Simulation results illustrate the effectiveness of the proposed method, which provides more decision-making information to perform the subsequent analytic for integrated electricity-gas-heat energy system.

CRediT authorship contribution statement

Yujia Huang: Methodology, Software, Data curation, Writing – original draft. **Qiuye Sun:** Conceptualization, Supervision, Funding acquisition. **Yushuai Li:** Visualization, Investigation, Writing – review & editing. **Chenghao Sun:** Software, Writing – review & editing. **Zhe Chen:** Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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