

The Small-Signal Stability Analysis of the Droop-Controlled Converter in Electromagnetic Timescale

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Abstract—Recently, the converters controlled by droop controller with phase-locked loop were observed in islanded ac microgrids. However, only the state-space-based approach was applied to investigate the stability of this converter in the previous works. Compared with the state-space-based approach, the characteristic equation approach has plenty of advantages, such as convenient stability margin analysis (phase margin, gain margin, etc.) and simple stability criterion (Routh criterion). Thus, a novel small-signal modeling approach based on characteristic equation for converter-dominated ac microgrids is proposed to assess the system low-frequency stability in this paper. First, considering zero-order holder and time delay, the small-signal characteristic equation of this converter is presented by Padé approximation and dynamic phasor model. Furthermore, the implementation and parameter design of the converters are studied under the practical considerations. Compared with the existing characteristic equation methods, the proposed approach can verify that the performance is significantly improved. Eventually, simulations and experimental results are presented, indicating that the proposed approach can assess the system low-frequency stability conveniently and accurately.

Index Terms—Small-signal model, converter, droop control, characteristic equation.

I. INTRODUCTION

WITH the rapid development of the microgrids consisting of the high penetration of renewable energy resources, the power electronic converters have been widely applied to microgrids [1]. Meanwhile, the stability assessment of microgrids dominated by power electric converters has been an attractive issue [2]. Traditionally, there are three common stability problems, i.e., static stability, dynamic stability and transient stability. The latter two pay more attention to large-signal perturbation, such as load variations and faults. While the former focuses on small-signal disturbance. The static stability is studied by small-signal method in electromagnetic timescale

[3]–[4]. Meanwhile, the dynamic stability and transient stability problems should be researched by large-signal method. Compared with the traditional power system, the small-signal stability of converter is a critical issue due to the low-inertia and negative-impedance nature of systems dominated by power electric converters [5]. Numerous small-signal stability issues have been introduced in electromagnetic timescale, such as low-frequency oscillation (LFO) and harmonic oscillation (HO) [3], [5]. From the perspective of multi-timescale, stability assessment can be divided into three parts [3], and from the point of view of frequency domain in electromagnetic timescale, LFO and HO needed to be separately researched [6]. Wherein, the LFO events have occurred worldwide in different kinds of traction power supply systems [6]. Thus, this paper focuses on stability analysis of LFO in converter-dominated islanded ac microgrids.

Obviously, power sharing based on droop control was widely used in paralleled converters in electromagnetic timescale [7]–[9]. In order to achieve seamless transition between grid-tied operation and islanded operation, the converter controlled droop controller with phase-locked loop (PLL) in grid-tied/islanded microgrid has become an advisable choice [10]–[11]. Meanwhile, in order to improve the performance, numerous modified structures of the PLL were widely investigated [12]–[14]. According to the literature [14], there are two main improved structures, i.e., PLL structure based on transport delay with phase error compensation and orthogonal system generator SOGI, respectively. Furthermore, regardless of the PLL structure chosen, a small signal approach was pursued [14]. From the viewpoint of the stability analysis, the state-space-based approach was proposed to assess the stability of the converter controlled by droop controller with/without PLL in islanded ac microgrid [15]–[17]. However, characteristic equation approach had plenty of advantages. Therefore, this paper focuses on the small-signal modeling approach based on characteristic equation for the converter controlled by droop controller with/without PLL in islanded ac microgrid to assess the system stability.

Recently, three main types of approaches were used to analyze the stability of the converters, i.e., impedance-based approach, state-space-based approach and characteristic equation approach. Impedance-based approach, based on the ratio of the impedances of two subsystems at the interface, had several potential advantages to clearly illustrate the impact of individual components or subsystems on the system stability [4], [18]–[20].

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Meanwhile, this approach was more concerned with (i) the interactive stability of a grid-connected inverter and grid; (ii) the stability analysis of the whole microgrid consisting of multi-DGs. However, the stability of the single converter cannot be analyzed. Furthermore, it was significant to check the right-half-plane (RHP) poles of the transfer function matrix when using the generalized Nyquist criterion (GNC) for stability analysis [18]. Thus, a large number of papers were proposed to modify GNC, such as Inverse Nyquist stability criterion [4]. Additionally, apparent impedance analysis and characteristic analysis of subsynchronous resonance were proposed to assess the stability in different cases [19]–[20]. To reduce the computational burden, impedance specifications and norm criteria were proposed one after another [21]–[22]. Both state-space-based approach and characteristic equation approach are well established in investigating the stability of the single converter. State-space-based approach was a global stability analysis method to assess the stability of microgrids regardless of the location of the instability source [15]–[17], [23]. This approach always adopted linearized method around an equilibrium point [16]–[17]. Nevertheless, the 15th order mathematical model of one inverter, including the controllers and the filter, was used to facilitate the dynamical analysis and the eigenvalue researches. Thus, the time domain analysis of a microgrid system with multiple inverters was still time-consuming, and it was a computational burden. Therefore, the necessity of the reduced-order modeling of multi-inverters could never be more obvious [16]. The Kron reduction method and the singular perturbation method were broadly utilized to deal with above-mentioned time-consuming and complex problem [16]–[17], [23]. Compared with state-space-based approach, the characteristic equation approach has plenty of advantages, such as stability margin analysis and simply stability criterion (Routh Criterion). There are three main types of stability margin analysis methods, i.e., Phase Margin (PM), Gain Margin (GM) and σ_1 Margin. It is more convenient for designers to assess the system stability margin by using characteristic equation approach. From the viewpoint of the stability margin analysis and stability criterion, the characteristic equation approach is a better choice. Meanwhile, the reduced-order method of characteristic equation approach was also an important issue [24]–[25]. However, the ZOH and time delay were not considered, which would result in the inaccurate stability assessment.

In view of the aforementioned problems, this paper proposes a novel small-signal modeling approach based on characteristic equation for converter-dominated ac microgrid. The main features and benefits of this paper are listed as follows:

- 1) Within the context of the droop controller with PLL, a novel small signal modeling approach based on characteristic equation is first proposed to assess the stability of the converter-dominated islanded ac microgrid;
- 2) Within the context of the droop controller without PLL, the time delay, ZOH and dynamic phase are embedded into the proposed stability analysis method, and the performance of the proposed method is better than that in literatures [24]–[25];
- 3) Combining with impedance-based method and inner voltage/current controller bandwidth, the proposed stability

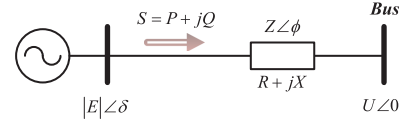


Fig. 1. Standalone mode model.

analysis method can provide the necessary and sufficient condition to assess the stability of the complex microgrid.

The rest of this paper is organized as follows. This paper proposes a novel small-signal modeling approach based on characteristic equation for converters controlled by droop controller with/without PLL in Section II. Section III provides parameters selection scheme of converters. In Section IV and Section V, the proposed small-signal modeling approach is validated through extensive simulation and experimental results. Finally, the conclusion is obtained in Section VI.

II. SMALL-SIGNAL MODELING OF THE CONVERTER WITH/WITHOUT PLL

As shown in Fig. 1, the relationship between the active or reactive power and the line impedance can be illustrated by using a bus system with a distributed generation (DG) unit and a slack bus. The power flow through the line can be expressed as:

$$\begin{cases} P = \frac{3}{Z^2} (RE^2 - REU \cos \delta + XEU \sin \delta) \\ Q = \frac{3}{Z^2} (XE^2 - XEU \cos \delta - REU \sin \delta), \end{cases} \quad (1)$$

where P and Q are the active and reactive power flowing from DG to the slack bus, respectively; $Z = R + j\omega L = R + jX$ is the impedance of line between DG and the slack bus; δ is the bus angle in DG; E and U are the voltage magnitudes of the DG and the slack bus, respectively. Only when the line impedance is mainly inductive ($R \approx 0$), the power flowing from DG to the slack bus can be rewritten as [26]:

$$\begin{cases} P = \frac{EU}{X} \sin \delta \\ Q = \frac{E(E - U \cos \delta)}{X}. \end{cases} \quad (2)$$

Furthermore, for $\delta \approx 0$, it can be found that $\sin \delta \approx \delta$ and $\cos \delta \approx 1$. The local control strategy of the DG (the well-known droop control strategy ($P - \omega$ and $Q - E$)) in an inductive network can be written as

$$\begin{cases} \omega = \omega^* - m(P - P^*) \\ E = E^* - n(Q - Q^*), \end{cases} \quad (3)$$

where P^* and Q^* are the active and reactive power references, respectively. ω^* and E^* are the angular velocity and voltage references, respectively. In [16] and [17], Fig. 2 shows the overall control strategy of the converters controlled by droop controller with PLL. According to the definition of a mirror frequency decoupled (MFD) in [27], the PLL and active as well as reactive power controllers are not mirror frequency decoupled (NMFD)

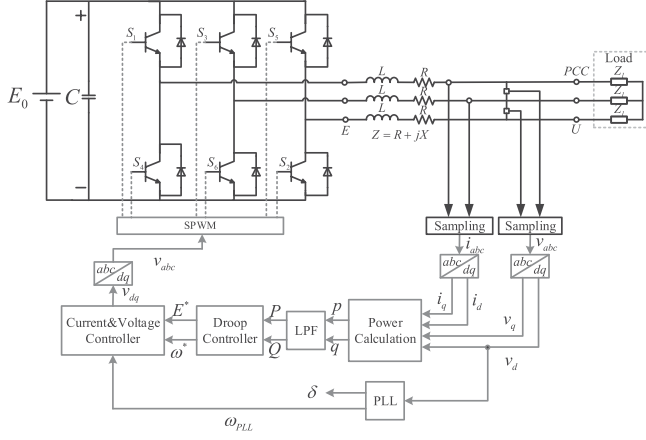


Fig. 2. The overall control strategy of DG adopting the droop controller with PLL.

system. If a converter is incorrectly supposed to be MFD, it will result in an inaccurate or erroneous evaluation of the characteristic equation. As a result, dynamic phasor model (DPM) should be considered, and the behaviour of a dynamical system around an equilibrium point can be qualitatively linearized [25]. Thus, for NMFD system, the generalized modeling based on complex time domain waveform $x(t)$ can be obtained within the interval $\tau \in (t - T, t]$ by Fourier series.

$$x(\tau) = \sum_{k=-\infty}^{\infty} X_K(t) e^{jk\omega_s \tau}, \quad (4)$$

where $\omega_s = 2\pi/T$, $X_K(t)$ are the complex Fourier coefficients, respectively. While the dynamic k^{th} phasor at time t , $X_K(t)$ can be given based on integral form within the time interval.

$$X_K(t) = \frac{1}{T} \int_{t-T}^t x(\tau) e^{-jk\omega_s \tau} d\tau = \langle x \rangle_k(t), \quad (5)$$

where $\langle x \rangle_k(t)$ is the average k^{th} phase during the whole period T . The important characteristic of the phasors is derived with respect to the time of the k^{th} dynamic phasor X_k , which can be represented as follows:

$$dX_K(t)/dt = \langle dx/dt \rangle_k(t) - jk\omega_s X_K(t). \quad (6)$$

To sum up, for example, the relationship between an inductor voltage U and its current i can be expressed as follows:

$$U = L \frac{di}{dt} + j\omega L i. \quad (7)$$

Thus, the linearized equation of the equation (3) as well as the power flow in output filter represented in [28] can be rewritten as follows:

$$\begin{cases} \Delta\omega = \Delta\omega^* - m\Delta P + m\Delta P^* \\ \Delta E = \Delta E^* - n\Delta Q + n\Delta Q^* \\ \Delta P = m_{11}\Delta E + m_{12}\Delta\phi \\ \Delta Q = n_{11}\Delta E + n_{12}\Delta\phi, \end{cases} \quad (8)$$

where

$$\begin{cases} m_{11} = \frac{3(Ls + R)E}{(Ls + R)^2 + (\omega L)^2} \\ m_{12} = \frac{3\omega L E^2}{(Ls + R)^2 + (\omega L)^2} \\ n_{11} = \frac{3\omega L E}{(Ls + R)^2 + (\omega L)^2} \\ n_{12} = \frac{-3(Ls + R)E^2}{(Ls + R)^2 + (\omega L)^2}. \end{cases} \quad (9)$$

Moreover, the recent researches illustrate that the time delay and ZOH regarding the digital control and sampling system may also result in instability. Nevertheless, such effects of time delay [29] and ZOH [30] are often neglected in the existing literatures based on characteristic equation. Thus, a novel small-signal modeling approach based on characteristic equation considering the time delay and ZOH of the digital control and sampling system would be researched. The *Padé* approximation is a method of approximating rational functions which is superior to Taylor expansion. And it is always applied to equivalent the time delay and ZOH, where the exponential term can be approximated as a polynomial function as shown in (10).

$$e^{-\tau s} = \frac{b_0 + b_1 + \dots + b_i(\tau s)^i + \dots + b_l(\tau s)^l}{a_0 + a_1 + \dots + b_j(\tau s)^j + \dots + b_k(\tau s)^k}, \quad (10)$$

where $a_j = \frac{(l+k-j)!k!}{j!(k-j)!}$, $j = 0, \dots, k$, $b_i = (-1)^i \frac{(l+k-i)!l!}{i!(l-i)!}$ and $i = 0, \dots, l$.

Considering the time delay of one period of the ac microgrid frequency, and ZOH time of half a period of the ac microgrid frequency [29], and by utilizing the first-order *Padé* approximation, the time delay and ZOH can be respectively rewritten as

$$e^{-\tau_t} \approx \frac{2 - Ts}{2 + Ts}, \quad (11)$$

$$e^{-\tau_z} \approx \frac{4 - Ts}{4 + Ts}, \quad (12)$$

where T is the period of the ac microgrid frequency, τ_t and τ_z are the delay time and ZOH time.

Considering the characteristics of the converters controlled by droop controller with PLL, the PLL would also become a main factor to result in an instable phenomenon in the converter-based ac microgrids [1], [3], [31]. The PLL parameters (mainly the PI gain) are varied to decide the potential instability. In other words, it is sure that the pole trajectories of the converter-based ac microgrids will immigrate the right-half of the complex plane with variations of the PI gains. Further, when high-bandwidth PLL parameters are decided, a fast phase tracking performance can be achieved, whilst, a high-bandwidth will also degrade the tracking ability of the PLL with unbalanced operating conditions. Meanwhile, when low-bandwidth PLL parameters are decided, the electromechanical oscillation modes may be affected. In general, the PLL-tracked phase is considered as the d-axis direction of the d-q coordinates, as shown in Fig. 3. Thus,

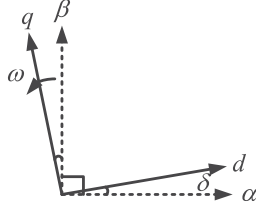


Fig. 3. Phase tracking of the terminal voltage by the PLL.

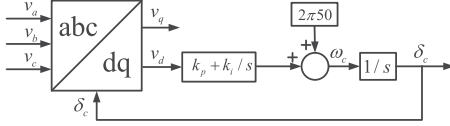


Fig. 4. The synchronous-reference-frame PLL.

it can be seen from Fig. 4 that the dynamic of the PLL should be considered [1], which indicates that the three-phase space-phasor angular velocity can be described as

$$\omega = G_{PLL}(s) V_c, \quad (13)$$

where $G_{PLL} = k_p s + k_i/s^2 + s + k_p$, which can be obtained by the Eq.(1) in the literature [1].

Remark 1: The structure of the PLL provided in this paper is a widely adopted structure [16]-[17], [32]. Of course, the PLLs are also widely modified to improve performance. According to the literature [14], there are two main improved structures, i.e., PLL structure based on transport delay with phase error compensation and orthogonal system generator second order generalized integrator (SOGI) [14], respectively. As mentioned in literature [14], “Regardless the PLL structure chosen, a small signal approach has been pursued”. The proposed small-signal analysis can be applied to assess the stability of the inverters with notable differences in terms of dynamics by using a first order Taylor expansion and *Padé* approximation. It is undeniable that parts of the characteristics of PLL will not be reflected due to simplification.

Additionally, the inner loop controllers of the converter need to provide active or passive damping for possible oscillations between the output filters of the converters and the transmission line. Thus, a low-pass filter should be applied to converters, and the active and reactive output powers obtained by the measuring block are given by (3). Therefore, the average values of the P and Q considering time delay as well as ZOH can be given by:

$$\begin{cases} p = \frac{\omega_f}{s + \omega_f} e^{-\tau_z} P e^{-\tau_t} \\ q = \frac{\omega_f}{s + \omega_f} e^{-\tau_z} Q e^{-\tau_t}, \end{cases} \quad (14)$$

where ω_f is the cutoff frequency and s is the Laplace operator [16]-[17].

The angular velocity can be expressed as

$$\Delta\omega = s\Delta\phi \quad (15)$$

By combining (3)-(15), the characteristic equation of the droop controller with PLL is obtained as follows:

$$\begin{aligned} &\alpha s^{11} + \beta s^{10} + \gamma s^9 + \epsilon s^8 + \varepsilon s^7 + \zeta s^6 \\ &+ \eta s^5 + \vartheta s^4 + \iota s^3 + \kappa s^2 + \lambda s + \mu = 0, \end{aligned} \quad (16)$$

The parameters can be found in Appendix. From the view-point of simplification, the order of characteristic equation of the droop controller with PLL can be reduced by ignoring time delay and ZOH. Of course, this small-signal modeling approach can also be applied to the converters controlled by droop controller without PLL. Wherein, the characteristic equation can be shown as follows:

$$\xi s^7 + o s^6 + \pi s^5 + \varpi s^4 + \rho s^3 + \sigma s^2 + \varsigma s + \tau = 0, \quad (17)$$

The parameters can be found in Appendix. Considering converters controlled by droop controller without PLL, it should be noted that Coeiho et al proposed an order-reduced method in 1999, which is shown in equation (18) [24]. Furthermore, the dynamic phasors-based model (DPM) is proposed in 2014, which was shown in equation (19) [25]. These are two common methods to assess the converter stability based on the characteristic equation. The parameters can be found in Appendix.

$$a_1 s^3 + b_1 s^2 + c_1 s + d_1 = 0, \quad (18)$$

$$a_2 s^5 + b_2 s^4 + c_2 s^3 + d_2 s^2 + e_2 s + f_2 = 0, \quad (19)$$

Remark 2: Although the proposed approach based on droop controller with PLL shown in (16), seems to be complex, the whole process can be significantly simplified by MATLAB. When the time delay, ZOH, dynamic phasor model and PLL are not considered, the characteristic equation of the converter can be seen as Eq. (18). Thus, when the time delay, ZOH, dynamic phasor model and PLL are considered, the characteristic equation of the converter can be obtained by variable substitution and MATLAB, i.e., $R = R + sL$, $\omega = G_{PLL}(s) E$, $m = (4 - 3T)m/(4 + 3Ts)$ and $n = (4 - 3T)n/(4 + 3Ts)$. Therefore, the characteristic equation mathematical derivation is not difficult. Furthermore, compared with the DPM approach shown in (19), the order of the proposed approach based on droop controller without PLL, which is shown in (17), is only raised by two orders, meanwhile, the performance of this approach is significantly improved.

III. PARAMETERS SELECTION SCHEME

The parameters of three-phase voltage converters controlled by droop controller without PLL have been widely researched [17], whereas the PLL parameters of converters controlled by droop controller with PLL should be studied. Fig. 4 shows the typical PLL structure, and the closed-loop transfer function is given by (20) [1].

$$G_{PLL} = \frac{k_p s + k_i}{s^2 + s + k_p} \quad (20)$$

The PLL PI parameters shown in (20) can be represented in light of the ω_n and ζ , where $k_p = 2\zeta\omega_n$ and $k_i = \omega_n^2$, the natural frequency ω_n can be chosen as the indicator of the PLL

TABLE I
THE SIMULATION SYSTEM PARAMETERS IN CASE A

Parameters	Values	Parameters	Values
DC link voltage	600V	Rate microgrid frequency	50Hz
Link inductance	3mH	Link resistor	1 Ω
Output voltage	220V/50Hz	Low pass filter frequency	5Hz
Droop coefficient m	$1e-5$	Droop coefficient n	$3e-4$

bandwidth, and ζ is the damping factor. Furthermore, the boundary of dc-side capacitor can be obtained in light of literature [33].

The main contributors to the harmonic oscillation in any converter based generation installation are the inner voltage and current control loops. According to the literatures [34]-[35], the ac voltage control loop bandwidth is around 5 times lower than the inner-loop current controller and that satisfies the standard bandwidth ratio design criterion to ensure that system is expected to operate stably. Thus, the stability of the inner voltage/current controller can be ensured in advance by tuning their bandwidth to satisfy the standard bandwidth ratio design criterion.

IV. VALIDATION BY NUMERIC SIMULATION

To validate the performance of the proposed small-signal modeling approach based on characteristic equation for the islanded ac microgrid, converters in various control conditions have been simulated. Meanwhile, in order to eliminate the impact of the source subsystem load, the impedance is zero at the dc grid side (short circuit ratio is infinite). Moreover, the small disturbance source in this paper is an external high-bandwidth small-signal sinusoidal perturbation current which is same with that in our previous literature [36] and is injected into the dc bus. Furthermore, the inner-loop current controller is tuned at $4 + 800/s$ and the close-loop control bandwidth is 160 Hz, and inner-loop voltage controller is $1 + 3/s$ and the control bandwidth is 7.2 Hz. Thus, the control tunings satisfy the standard bandwidth requirements and the inner current/voltage controller is expected to operate stably.

A. The Converter Controlled by Droop Controller Without PLL

Section II presents the proposed small-signal modeling approach depicted in (17), the conventional reduce-order small-signal modeling depicted in (18) and DPM approach depicted in (19). The system parameters used in this analysis are shown in Table I. Compared with the conventional reduce-order small-signal modeling [24], the performance of DPM approach has been notably improved [25]. Thus, the performance of comparison between DPM approach and proposed approach should be just achieved. The eigenvalues of the characteristic equation by using different stability analysis methods are shown in Table II and Table III, respectively. As shown in Table II, there is no right zero of the characteristic equation based on the proposed approach, whereas, there are two right zeros of the characteristic equation based on the DPM approach. From the eigenvalues of the characteristic equation point of view, the system should be stable by utilizing the

TABLE II
THE EIGENVALUES OF THE CHARACTERISTIC EQUATION BASED ON THE PROPOSED APPROACH IN CASE A

λ_1	λ_2	λ_3	λ_4	λ_5	$\lambda_{6/7}$
-0.858	-132.253	-150.8	-40.353	-29.232	$-337.986 \pm j315.58$

TABLE III
THE EIGENVALUES OF THE CHARACTERISTIC EQUATION BASED ON THE DPM APPROACH IN CASE A

λ_1	$\lambda_{2/3}$	$\lambda_{4/5}$
-111111.108	$-0.909 \pm j0.321$	$0.907 \pm j1.314$

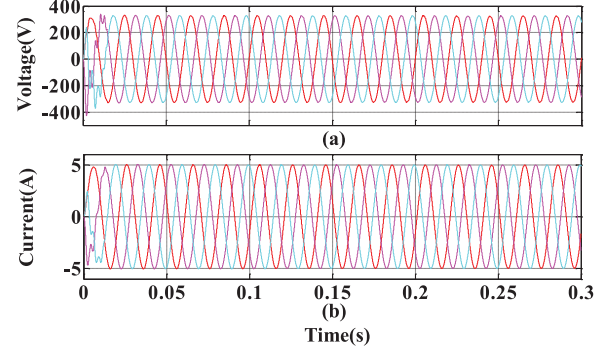


Fig. 5. Simulation of converter without PLL output behavior. (a) Output voltage. (b) Output current.

TABLE IV
THE EIGENVALUES OF THE CHARACTERISTIC EQUATION BASED ON THE PROPOSED APPROACH WHEN THE PARAMETERS OF THE CONVERTER IS CHANGED IN CASE A

λ_1	$\lambda_{2/3}$	$\lambda_{4/5}$	$\lambda_{6/7}$
-198.831	$-368.292 \pm j334.684$	$-58.165 \pm j94.358$	$11.139 \pm j44.665$

TABLE V
THE EIGENVALUES OF THE CHARACTERISTIC EQUATION BASED ON THE DPM APPROACH WHEN THE PARAMETERS OF THE CONVERTER IS CHANGED IN CASE A

λ_1	$\lambda_{2/3}$	$\lambda_{4/5}$
-111111.108	$-3.633 \pm j3.538$	$3.632 \pm j3.761$

proposed approach, whereas, the system is unstable in light of the DPM approach. The stability of the converter can be clearly verified by testing the time-domain waveforms. Apparently enough, the simulation system is stable as shown in Fig. 5 where the ac-side voltage/current are in accordance with the stable phenomenon. Therefore, compared with exciting literature based on characteristic equation approach, the effectiveness of the proposed approach is notably improved. In order to further validate the effectiveness of the method, the droop coefficient parameters of the converter are changed from $m = 1 \times 10^{-5}$ and $n = 3 \times 10^{-4}$ to $m = 1 \times 10^{-3}$ and $n = 3 \times 10^{-3}$, and inner-loop voltage controller is $10 + 100/s$. As shown in Tables IV and V, the system should be unstable by utilizing the proposed approach and DPM approach. The stability of the converter can be clearly verified by testing the time-domain waveforms. Apparently enough, the simulation system is unstable as shown in Fig. 6 where the ac-side voltage/current

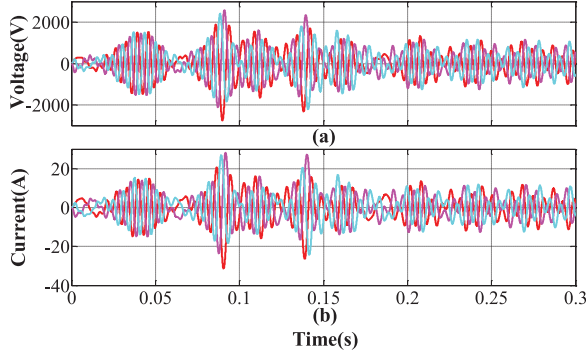


Fig. 6. Simulation of converter without PLL output behavior when the parameters of the converter is changed. (a) Output voltage. (b) Output current.

TABLE VI
THE SIMULATION SYSTEM PARAMETERS IN CASE B

Parameters	Values	Parameters	Values
DC link voltage	600V	Rate microgrid frequency	50Hz
Link inductance	4.2mH	Link resistor	0.5Ω
Output voltage	220V/50Hz	Low pass filter frequency	50.26rad/s
Droop coefficient m	$1e-4$	Droop coefficient n	$1e-4$

TABLE VII
THE EIGENVALUES OF THE CHARACTERISTIC EQUATION WITH
 $k_p = 0.25$ AND $k_i = 0.1$ IN CASE B

λ_1	λ_2	λ_3	$\lambda_{4/5}$
-0.402	-16.929	-180.045	-2.439±j4.542
$\lambda_{6/7}$	$\lambda_{8/9}$	$\lambda_{10/11}$	
-67.762±j43.915	-142.985±j45.530	-0.495±j0.028	

are in accordance with the unstable phenomenon. Thus, the performance of the proposed stability analysis method can be verified.

B. The Converter Controlled by Droop Controller With PLL

In this subsection, the effectiveness of the proposed approach for the converter controlled by droop controller with PLL is validated by two groups of simulations. Firstly, the system parameters are similar with that in literature [17] shown in Table VI, and PLL parameters are $k_p = 0.25$ and $k_i = 0.1$. The eigenvalues of the characteristic equation can be shown in Table VII, and there is no right zero of the characteristic equation based on the proposed approach, which indicates the simulation system should be stable. The stability of the converter can be clearly verified by testing the time-domain waveforms. At $t = 0$ s, the load is set as $P_{load} = 1.3$ kW, $Q_{load} = 0.5$ kVar. Furthermore, at $t = 0.1$ s, the load is set as $P_{load} = 1.8$ kW, $Q_{load} = 0.5$ kVar, and $t = 0.2$ s, the converter switches from islanded to grid-tied mode. As shown in Fig. 7, the output voltage/current of the simulation system are always stable, which is in accordance with the proposed approach. Secondly, in order to explain the influence of PLL parameters, the PLL parameters are only changed from $k_p = 0.25$ and $k_i = 0.1$ to $k_p = 0.25$ and $k_i = 10$. The eigenvalues of the characteristic equation can be shown in Table VIII, and there are two right zeros ($\lambda_{10/11} = 3.506 \pm j7.354$), which indicates the simulation result should be unstable. The oscillation frequency is 1.17 Hz, and damping factor is 0.43. The

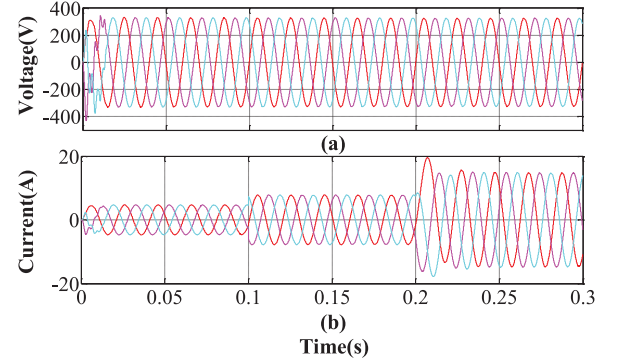


Fig. 7. Simulation of converter with PLL output behavior ($k_p = 0.25$ and $k_i = 0.1$). (a) Output voltage. (b) Output current.

TABLE VIII
THE EIGENVALUES OF THE CHARACTERISTIC EQUATION WITH
 $k_p = 0.25$ AND $k_i = 10$ IN CASE B

λ_1	λ_2	λ_3	$\lambda_{4/5}$
-180.047	-1.455	-0.165	-67.715±j43.851
$\lambda_{6/7}$	$\lambda_{8/9}$	$\lambda_{10/11}$	
-142.980±j45.538	-14.347±j5.791	3.506±j7.354	

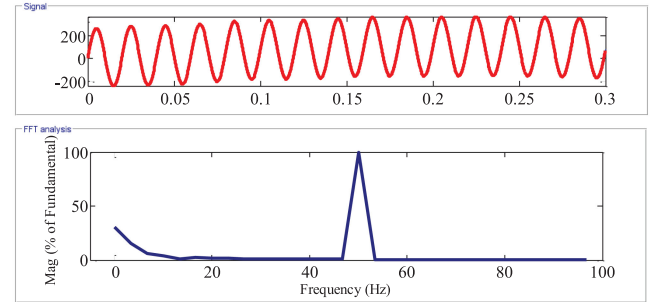


Fig. 8. Simulation of converter with PLL output behavior ($k_p = 0.25$ and $k_i = 10$).

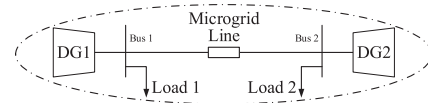


Fig. 9. The classic microgrid test system [8], [16].

stability of the converter can be clearly verified by testing the time-domain waveforms. Apparently enough, the simulation result which is shown in Fig. 8, is unstable, which is in accordance with the proposed approach. Thus, the simulation results demonstrate the effectiveness of the proposed approach.

C. The Classical Islanded AC Microgrid

In this subsection, the performance of the proposed approach for the classical islanded ac microgrid, which is shown in Fig. 9, is validated by two groups of simulations. The parameters are same as that in Case B. To analyze, it has been considered that the active/reactive droop gain of DG1 is two times bigger than that of DG2. According to the proposed approach, the eigenvalues of the characteristic equation can be shown in

TABLE IX
THE EIGENVALUES OF THE CHARACTERISTIC EQUATION WITH
 $k_p = 0.25$ AND $k_i = 0.1$ IN CASE C

λ_1	λ_2	λ_3	λ_4
-0.402	-16.929	-180.045	-0.404
λ_5	λ_6	$\lambda_{7/8}$	$\lambda_{9/10}$
-2.578	-194.717	$-2.439 \pm j4.542$	$-67.762 \pm j43.915$
$\lambda_{11/12}$	$\lambda_{13/14}$	$\lambda_{15/16}$	$\lambda_{17/18}$
$-142.985 \pm j45.530$	$-0.495 \pm j0.028$	$-1.611 \pm j16.376$	$-61.851 \pm j60.852$
$\lambda_{19/20}$	$\lambda_{21/22}$		
$-149.559 \pm j61.757$	$-0.497 \pm j0.020$		

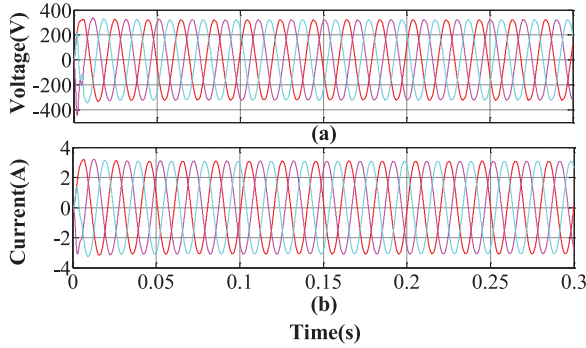


Fig. 10. Output behavior of the bus 1 ($k_p = 0.25$ and $k_i = 0.1$). (a) Output voltage. (b) Output current.

TABLE X
THE EIGENVALUES OF THE CHARACTERISTIC EQUATION WITH
 $k_p = 0.25$ AND $k_i = 10$ IN CASE C

λ_1	λ_2	λ_3	λ_4
-180.047	-1.455	-0.165	-194.718
λ_5	λ_6	$\lambda_{7/8}$	$\lambda_{9/10}$
-1.090	-0.224	$-67.715 \pm j43.851$	$-142.980 \pm j45.538$
$\lambda_{11/12}$	$\lambda_{13/14}$	$\lambda_{15/16}$	$\lambda_{17/18}$
$-14.347 \pm j5.791$	$3.506 \pm j7.353$	$-61.796 \pm j60.822$	$-149.559 \pm j61.765$
$\lambda_{19/20}$	$\lambda_{21/22}$		
$-4.335 \pm j2.709$	$1.336 \pm j0.844$		

Table IX, and there is no right zero of the characteristic equation based on the proposed approach, which indicates that the simulation system should be stable. The stability of the converter can be clearly verified by testing the time-domain waveforms. As shown in Fig. 10, the output voltage/current of the simulation system are always stable, which is in accordance with the proposed approach. Secondly, in order to explain the influence of PLL parameters, the PLL parameters are only changed from $k_p = 0.25$ and $k_i = 0.1$ to $k_p = 0.25$ and $k_i = 10$. By utilizing the proposed approach, the eigenvalues of the characteristic equation can be shown in Table X, and there are four right zeros ($\lambda_{10/11} = 3.506 \pm j7.354$, $\lambda_{21/22} = 1.336 \pm j0.844$) of the characteristic equation, which indicates that the simulation results should be unstable. The oscillation frequencies are 1.17 Hz and 1.73 Hz, and damping factors are 0.43 and 0.12. The stability of the converter can be clearly verified by testing the time-domain waveforms. Apparently enough, the simulation result which is shown in Fig. 11, is unstable, which is in accordance with the proposed approach. Thus, the simulation results demonstrate the effectiveness of the proposed approach.

For complex microgrid, the stability assessment can be divided into three steps. (i) According to the literatures [34]-[35],

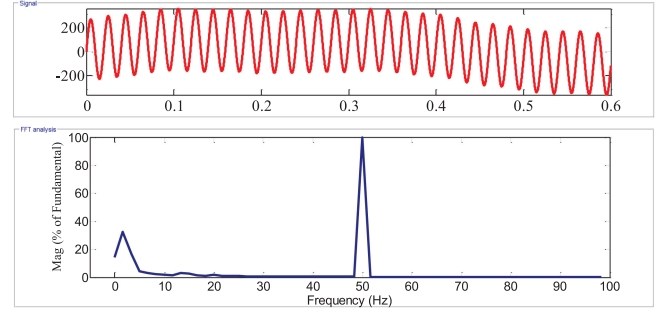


Fig. 11. Output behavior of the bus 1 ($k_p = 0.25$ and $k_i = 10$).

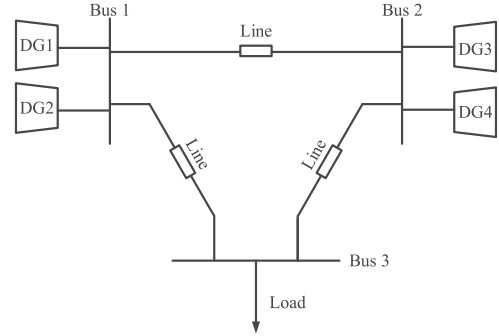


Fig. 12. The multiple inverters based microgrid.

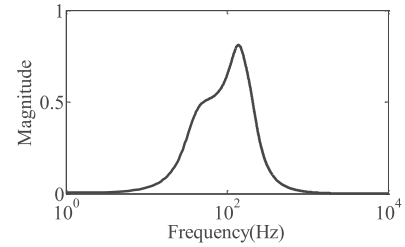


Fig. 13. The infinity-norm of impedance matrix.

the stability of the inner voltage/current controller can be assessed by analyzing their control bandwidth; (ii) According to the proposed stability analysis method, the each converter can be judged; (iii) The complex microgrid can be assessed by impedance-based method [18], [22]. When the system parameters have been known, the model-based impedance method is a better choice [18]. However, for unknown parameters system, the measure-based impedance method will be an advisable choice [22]. A case in point is that the multiple inverters based microgrid can be shown in Fig. 12, the parameters of the converters are same with that in the first simulation in this Case. Thus, the Step (i) and Step (ii) can be satisfied, respectively. Furthermore, as shown in Fig. 13, the infinity-norm of impedance matrix is less than unit from 0 to 10000 Hz. Thus, the multiple inverters based microgrid should be stable. The stability of the multiple inverters based microgrid can be clearly verified by testing the time-domain waveforms. Thereinto, the simulation system is stable as shown in Fig. 14 where the ac-side voltage/current of bus 3 are in accordance with the stable phenomenon. Thus, the

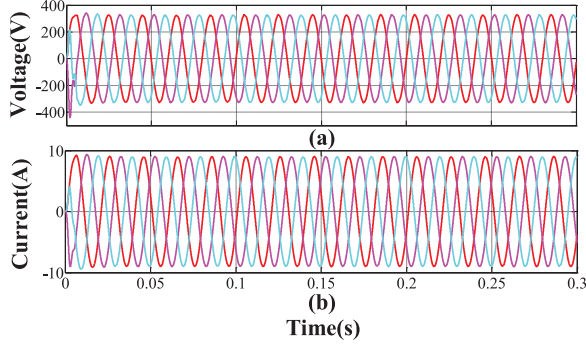


Fig. 14. Output behavior of the bus 3 of the multiple inverters based microgrid.

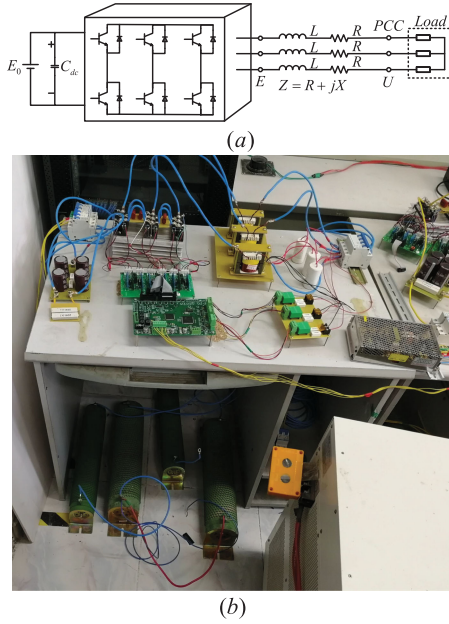


Fig. 15. Laboratory setup. (a) Schematic of the test system. (b) View of the test system.

effectiveness of the proposed stability analysis method can be verified.

V. EXPERIMENT RESULTS

A 110V, 50 Hz ac microgrid is built to verify the effectiveness of the proposed approach. As shown in Fig. 15, the test system consists of one three-phase inverter and one resistive load ($R = 40\Omega$). The whole platform of the microgrid is controlled by TMS320F28335DSP+XC6SLX9FPGA. Furthermore, the communication circuit is RS232&RS485, and the system switch frequency is 19.2 kHz. The system control parameters are shown in Table XI.

A. Stable Case

In this case, the parameters are shown in Table XI, and PLL parameters are $k_p = 0.25$ and $k_i = 1$. According to the proposed approach shown in (16), the eigenvalues of the characteristic equation can be shown in Table XII, and there is no right zero of the characteristic equation based on the proposed

TABLE XI
THE EXPERIMENTAL SYSTEM PARAMETERS

Parameters	Values	Parameters	Values
DC link voltage	200V	Rate microgrid frequency	50Hz
Link inductance	2mH	Link resistor	0.1 Ω
Output voltage	110V/50Hz	Low pass filter frequency	5Hz
Droop coefficient m	$1e-5$	Droop coefficient n	$3e-4$
Load	40 Ω	dc-side capacitor	1mF

TABLE XII
THE EXPERIMENTAL SYSTEM EIGENVALUES OF THE CHARACTERISTIC EQUATION WITH $k_p = 0.25$ AND $k_i = 1$

λ_1	λ_2	λ_3	λ_4
-0.179	-73.358	-143.020	-155.590
λ_5	$\lambda_6/7$	$\lambda_8/9$	$\lambda_{10/11}$
-1.227	$-0.67 \pm j1.59$	$-40.022 \pm j25.196$	$-5.022 \pm j5.141$

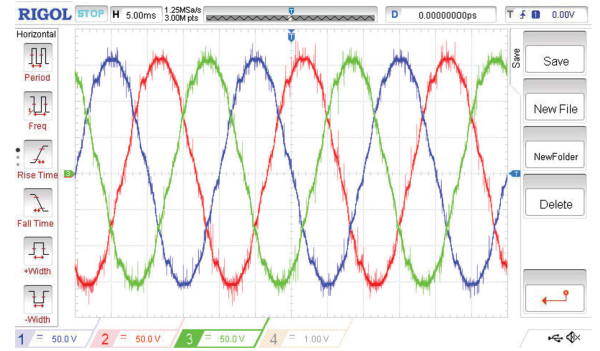


Fig. 16. Output voltage of converter adopting droop control with PLL ($k_p = 0.25$ and $k_i = 1$).

TABLE XIII
THE EXPERIMENTAL SYSTEM EIGENVALUES OF THE CHARACTERISTIC EQUATION WITH $k_p = 0.25$ AND $k_i = 10$

λ_1	λ_2	λ_3	λ_4
-155.583	-143.006	-73.667	-0.035
λ_5	$\lambda_6/7$	$\lambda_8/9$	$\lambda_{10/11}$
-8.488	$-39.855 \pm j25.520$	$-3.822 \pm j4.266$	$1.669 \pm j5.283$

approach, which indicates the experimental result should be stable. The stability of the converter can be conveniently verified by testing the time-domain waveforms. As shown in Fig. 16, the output voltage of experimental test system does not appear low-frequency instable phenomenon, which is in accordance with the proposed approach based on characteristic equation. Thus, the experimental result demonstrates the effectiveness of the proposed approach.

B. Unstable Case

In this case, the parameters are shown in Table XI. From the point of view of variable parameters, compared with stable case, only PLL parameters are changed from $k_p = 0.25$ and $k_i = 1$ to $k_p = 0.25$ and $k_i = 10$ in this case. According to the proposed approach, the eigenvalues of the characteristic equation can be shown in Table XIII, and there are two right zeros ($\lambda_{10/11} = 1.669 \pm j5.283$) of the characteristic equation based on the proposed approach, which indicates the experimental result should be unstable. The stability of the converter

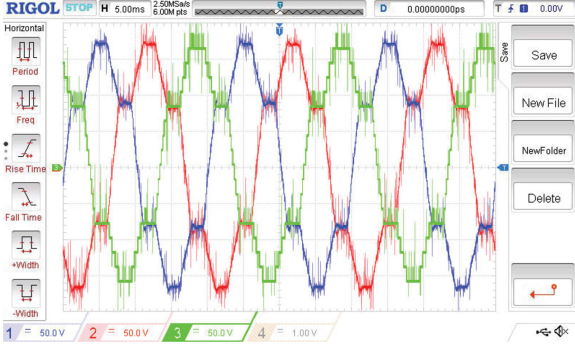


Fig. 17. Output voltage of converter adopting droop control with PLL ($k_p = 0.25$ and $k_i = 10$).

can be clearly verified by testing the time-domain waveforms. As shown in Fig. 17, the output voltage of the experimental test system appears instable phenomenon, which is in accordance with the proposed approach. Therefore, the experimental results demonstrate the effectiveness of the proposed approach.

VI. CONCLUSION

With the wide utilization of the converters controlled by droop controller with PLL, a novel small-signal modeling approach based on characteristic equation for converter-dominated islanded ac microgrid has been proposed to assess the system low-frequency stability. To improve the performance of the proposed approach, the time delay and ZOH have been considered by *Padé* approximation, and dynamic phasor model has been built by utilizing the definition of a MFD system. Furthermore, this approach could also be applied to the stability analysis of the converter controlled by droop controller without PLL. Compared with the existing characteristic equation method shown in [24]–[25], the effectiveness of the proposed approach could be notably improved. Combining with impedance-based method and inner voltage/current controller bandwidth, the proposed stability analysis method can provide the necessary and sufficient condition to assess the stability of the complex microgrid. Eventually, the effectiveness of the proposed approach based on characteristic equation for converters controlled by droop controller with PLL has been demonstrated by simulation and experimental results.

APPENDIX

The detail form of $\alpha \dots \mu$ in (16)

$$\begin{aligned}\alpha &= 4L^2T^2, \beta = 4LT(2LT\omega_f + 3L + 2RT + 2LT), \\ \gamma &= 4T^2(L^2 + R^2 + L^2k_p + 4L^2\omega_f + L^2\omega_f^2 + 4RL) \\ &\quad + 24LT(L + R + L\omega_f) + 9L^2, \\ \epsilon &= 6L^2(2T + 3\omega_f + 3 + 2T\omega_f^2 + 4Tk_p + 8T\omega_f) \\ &\quad + 12RT(R + 4L + 4L\omega_f) + 8T^2(R^2 + L^2k_p + L^2\omega_f \\ &\quad + R^2\omega_f + L^2\omega_f^2 + LR + 2LRk_p + 4LR\omega_f \\ &\quad + LR\omega_f^2 + 2L^2k_p\omega_f) + 18LR,\end{aligned}$$

$$\begin{aligned}\epsilon &= 8LRT(4Tk_p\omega_f + 2Tk_p + 2T\omega_f + 6k_p + 3\omega_f^2 \\ &\quad + 12\omega_f + 3) + 4L^2T(E^2Tk_p^2 + Tk_p^2 + 2Tk_p\omega_f^2 \\ &\quad + 4Tk_p\omega_f + T\omega_f^2 + 12k_p\omega_f + 6k_p + 6\omega_f^2 + 6\omega_f) \\ &\quad + (9L^2 + 4R^2T^2)(2k_p + \omega_f^2 + 4\omega_f + 1) \\ &\quad + 36RL(\omega_f + 1) + 3R^2(8T\omega_f + 8T + 4T^2 + 3) \\ &\quad + 12nE^2LT^2k_p\omega_f, \\ \zeta &= 4E^2T^2(9mnE\omega_f^2 + 3mELk_p\omega_f + 2L^2k_p^2\omega_f \\ &\quad + 2L^2k_i k_p) + 12E^2LT(Lk_p^2 - nTk_p\omega_f^2 - nTk_p\omega_f \\ &\quad - nTk_i\omega_f) + 4L^2Tk_p(2Tk_p\omega_f + 2T\omega_f^2 + 3k_p \\ &\quad + 6\omega_f^2 + 12\omega_f) + 6L^2\omega_f(2T^2\omega_f + 6k_p \\ &\quad + 3\omega_f + 3) + 8LRT^2(k_p^2 + 2k_p\omega_f^2 + 4k_p\omega_f \\ &\quad + \omega_f^2) + 48LRT(2k_p\omega_f + k_p + \omega_f^2 + \omega_f) \\ &\quad + 18LR(2k_p + \omega_f^2 + 4\omega_f + 1) + 8R^2T^2(2k_p\omega_f \\ &\quad + k_p + \omega_f^2 + \omega_f) + 6R^2(4Tk_p + 2T\omega_f^2 \\ &\quad + 8T\omega_f + 2T + 3\omega_f + 3) + 18L^2k_p, \\ \eta &= 12mE^3T\omega_f(6nT\omega_f - LTk_p\omega_f - LTk_p - LTk_i \\ &\quad - 9n\omega_f) + 4E^2L^2T(Tk_i^2 + 4Tk_i k_p\omega_f + Tk_p^2\omega_f^2 \\ &\quad + 6k_i k_p + 6k_p^2\omega_f) - 12nE^2LT^2\omega_f(k_i\omega_f + k_i \\ &\quad + k_p^2 + k_p\omega_f) + 9L^2(\omega_f^2 + E^2k_p^2 + 2k_p\omega_f^2 + 4k_p\omega_f \\ &\quad + k_p^2) + 16LRTk_p\omega_f(Tk_p + T\omega_f + 3\omega_f + 6) \\ &\quad + 12LR(2T\omega_f^2 + 6k_p\omega_f + 3k_p + 3\omega_f^2 + 3\omega_f) \\ &\quad + 4R^2T^2(k_p^2 + 2k_p\omega_f^2 + 4k_p\omega_f + \omega_f^2) + 24R^2T \\ &\quad (2k_p\omega_f + k_p + \omega_f^2 + \omega_f) + 9R^2(2k_p + \omega_f^2 \\ &\quad + 4\omega_f + 1) + 4L^2Tk_p\omega_f(Tk_p\omega_f + 6k_p + 6\omega_f) \\ &\quad + 27nE^2Lk_p\omega_f, \\ \vartheta &= 9mnE^3\omega_f^2(8T^2k_p + 4T^2 - 24T + 9) - 12mE^3LT^2 \\ &\quad \omega_f(k_i + k_p^2 + k_p + k_i\omega_f) + 4E^2L^2Tk_i(2Tk_i \\ &\quad \omega_f + 2Tk_p\omega_f^2 + 3k_i + 12k_p\omega_f) + 6E^2L^2k_p \\ &\quad (2Tk_p\omega_f^2 + 3k_i + 3k_p\omega_f) - 12nE^2LT^2\omega_f \\ &\quad (k_i k_p + k_i\omega_f + k_p^2\omega_f) + 27nE^2L\omega_f(k_i \\ &\quad + k_p\omega_f + k_p) + 8LRTk_p\omega_f(Tk_p\omega_f + 6k_p \\ &\quad + 6\omega_f) + 6L^2k_p\omega_f(2Tk_p\omega_f + 3k_p \\ &\quad + 3\omega_f) + 18LR(k_p^2 + 2k_p\omega_f^2 + 4k_p\omega_f + \omega_f^2) \\ &\quad + 4R^2Tk_p(2Tk_p\omega_f + 2T\omega_f^2 + 3k_p + 6\omega_f^2 \\ &\quad + 12\omega_f) + 6R^2(2T\omega_f^2 + 6k_p\omega_f + 3k_p \\ &\quad + 3\omega_f^2 + \omega_f) + 27mE^3Lk_p\omega_f,\end{aligned}$$

$$\begin{aligned}
\iota &= 27mE^3L\omega_f(k_i + k_p\omega_f + k_p)12mE^3LT^2\omega_f(k_i\omega_f \\
&\quad + k_p^2\omega_f + k_ik_p) + 18mnE^3\omega_f^2(4T^2k_p - 12Tk_p \\
&\quad - 6T + 9) + 4E^2L^2Tk_i\omega_f(Tk_i\omega_f + 6k_i + 6k_p\omega_f) \\
&\quad + 9E^2L(Lk_i^2 + 4Lk_pk_i\omega_f + 3n\omega_fk_i(\omega_f + 1) + Lk_p^2\omega_f^2 \\
&\quad + 3nk_p\omega_f(k_p + \omega_f)) + 12LRk_p\omega_f(2Tk_p\omega_f \\
&\quad + 3k_p + 3\omega_f) + 4R^2Tk_p\omega_f(Tk_p\omega_f + 6k_p + 6\omega_f) \\
&\quad + 9R^2(k_p^2 + 2k_p\omega_f^2 + 4k_p\omega_f + \omega_f^2) + 3k_p\omega_f^2(3L^2k_p \\
&\quad - 4nE^2LT^2k_i), \\
\kappa &= 27mE^3L\omega_f(k_i\omega_f + k_i + k_p^2 + k_p\omega_f) \\
&\quad + 9mnE^3\omega_f^2(4T^2k_p^2 + 24Tk_p + 18k_p + 9) + 6E^2Lk_i\omega_f \\
&\quad (2LTk_i\omega_f - 2mET^2k_p\omega_f + 3Lk_i + 3Lk_p\omega_f) \\
&\quad + 27nE^2L\omega_f(k_ik_p + k_i\omega_f + k_p^2\omega_f) \\
&\quad + 6Rk_p\omega_f(3Lk_p\omega_f + 2RTk_p\omega_f + 3Rk_p + 3R\omega_f), \\
\lambda &= 27mE^3\omega_f(L(k_ik_p + k_i\omega_f + k_p^2\omega_f) + 4nTk_p^2\omega_f \\
&\quad + 6nk_p\omega_f) + 9\omega_f^2(E^2L^2k_i^2 + 3nE^2Lk_ik_p + R^2k_p^2), \\
\mu &= 27mE^3k_p\omega_f^2(3nk_p + Lk_i).
\end{aligned}$$

The detail form of $\xi \dots \tau$ in (17)

$$\begin{aligned}
\xi &= 4L^2T^2, o = 4LT(2LT\omega_f + 3L + 2RT), \\
\pi &= 4L^2T(T\omega^2 + T\omega_f^2 + 6\omega_f) \\
&\quad + 4RT(4LT\omega_f + 6L + RT) + 9L^2, \\
\varpi &= 4L^2T(2T\omega^2\omega_f + 3\omega^2 + 3\omega_f^2) \\
&\quad + 2L\omega_f(9L + 4RT^2\omega_f + 24RT) \\
&\quad + 4T^2\omega_f(2R^2 - 3nEL\omega) + 6R(3L + 2RT), \\
\rho &= -12ELT^2\omega\omega_f(mE + n\omega_f) \\
&\quad + 4L^2T\omega^2\omega_f(T\omega_f + 6) + 9L^2(\omega^2 + \omega_f^2) \\
&\quad + 4RT\omega_f(6L\omega_f + RT\omega_f + 6R) + 9R(4L\omega_f + R), \\
\sigma &= 12mE^2T^2\omega_f^2(3nE - L\omega) \\
&\quad + 6R\omega_f(3L\omega_f + 2RT\omega_f + 3R) \\
&\quad + 3L\omega\omega_f(9nE + 4LT\omega\omega_f + 6L\omega), \\
\varsigma &= 27EL\omega_f(mE\omega + n\omega\omega_f - 4mnTE^2\omega_f) \\
&\quad + 9\omega_f^2(L^2\omega^2 + R^2), \\
\tau &= mE^2\omega_f^2(8nE + 27L\omega).
\end{aligned}$$

The detail form of $a_1 \dots d_1$ in (18)

$$\begin{aligned}
a_1 &= (R^2 + \omega^2L^2)^2, \\
b_1 &= 2(R^2 + \omega^2L^2)^2\omega_f + 3nRE(R^2 + \omega^2L^2), \\
c_1 &= 2(R^2 + \omega^2L^2)^2\omega_f^2 \\
&\quad + (R^2 + \omega^2L^2)(3mL\omega E^2 + 3nRE\omega_f^2),
\end{aligned}$$

$$d_1 = m\omega_f^2(9\omega^2L^2E^2 - 9nRLE^3\omega - 3RE^2(R^2 + \omega^2L^2)),$$

The detail form of $a_2 \dots f_2$ in (19) can be found in Eq.(35) in the literature [25].

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